TP1-A-1

A Serial Unequal Error Protection Code System using Multilevel Trellis Coded Modulation with Two Ring Signal Constellations for AWGN Channels

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Abstract— **A serial Unequal Error Protection (UEP) code system for use with information sources that contain a mixture of both important and less important data is proposed. To achieve UEP, the proposed scheme uses a form of Trellis Coded Modulation (TCM) which encodes the data by switching between two codes that use different signal constellations. The proposed system uses two ring signal constellations (2RING). No extra information about which code was used is added. The receiver decides which code was used by examining the received signal points. In this paper, a theoretical analysis of the 2RING signal constellation is presented, and the effectiveness of the system for AWGN channels is shown using computer simulations.**

I. INTRODUCTION

In order to protect information with different levels of importance, Unequal Error Protection (UEP) is useful. The advantage of using UEP codes rather than equal error protection codes is that bits that are deemed to be important can be protected more than bits of lesser importance. In many other communication applications, data with different levels of importance occurs. If a unique word transmitted in a data frame is mistakenly detected, this will cause errors in the following data, resulting in an error burst. Therefore, this unique word is more important than other data. Also, the data cell used in Asynchronous Transfer Mode (ATM) communication systems is one example. In the 53 bytes ATM cell, 5 bytes are used for the header information. The ATM header contains various control information and therefore should be protected more than the other 48 bytes. By using UEP, an increase in reliability and performance can be achieved. Another application is a combined digital satellite communication and human communication coding system [8]. Previously, UEP has been studied by several authors [1]-[7]. Therefore, we consider it is significant to study UEP systems.

To this end, we have studied a Trellis Coded Modulation (TCM) approach to UEP for Additive White Gaussian Noise (AWGN) channels [9],[10] and for frequency selective Rayleigh fading channels [11],[12] in previous work. However, we mainly presented simulation results in these papers.

In this paper, we study the TCM codes and theoretically analyze the overall proposed UEP system using the 2RING signal constellation. Moreover, we show the effectiveness of the proposed UEP system using computer simulations for AWGN channels.

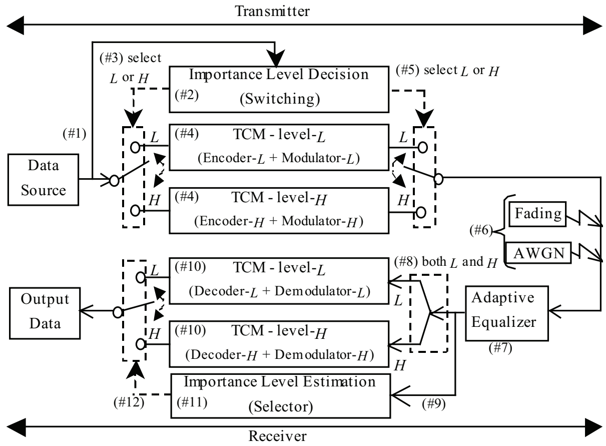

Fig.1 The proposed UEP scheme.

The rest of our paper is organized as follows. In section 2, we introduce our UEP scheme. In section 3, we present a theoretical analysis of our system. In sections 4, we present simulation results to show the performance of our system. Finally, we present our conclusions in section 5.

II. PROPOSED UEP SCHEME

A. System Model and Processing

The proposed system scheme is outlined in Fig.1. This scheme uses two importance levels. Hereafter, we describe the main processing steps used in the proposed scheme as shown in Fig.1. The numbers in the explanation, for example #1, correspond to the numbers in Fig.1.

Step1. The input data comes from an information source which outputs random data $\in \{0,1\}$. Also, the input data is transmitted to the importance level decision block (#1).

Step2. The importance level of the data is evaluated by the importance level decision block every N bits (#2). This decision controls the switch ahead. In this paper, in order to focus on the UEP implementation itself, we decide the importance level randomly. In Fig.1, if the output of the importance level decision block is "L", then the switches will be in the "up" position. If the output is "H", then the switches will be in the "down" position (#3).

Step3. We prepare codes which have different signal constellations according to the importance level. We describe this constellation in section 3.3 Moreover, we combine coding and modulation using TCM with four states [12] at the same time (#4). Information bits are allocated to signal points based on Gray coding. The distance between the signal points is larger for data-*H*, resulting in stronger error protection. Moreover, we encode the data differently according to the importance level to create UEP characteristics. The above-mentioned processing can be applied to the general case with M levels of importance. Here, in the proposed scheme, data-*H* is encoded for error protection using a rate 1/2, while data-*L* is not encoded.

Step4. The data is selected by changing the switches According to the importance level decision performed in step 2 every N bits (#5).

Step5. Multi-path fading in addition to AWGN affects the transmitted signal (#6). In the receiver, first we adopt an adaptive equalizer as a counter-measure for multi-path fading. However, we do not discuss the adaptive equalizer and fading (#7), because as we described in section 1, the focus of this paper is theoretical analyses of the proposed scheme in AWGN channels. Therefore in this paper, we do not consider the effect of fading in the channels.

Step6. The channel output is sent to all the decoders regardless of the importance (#8).

Step7. Also, the data is transmitted into the importance level estimator (#9). In parallel with the decoding (#10), the encoder used in the transmitter, i.e., the importance level, is estimated every N bits using a importance level estimation algorithm based on Maximum Likelihood Detection (MLD) (#11). Processing #10 and #11 are done in parallel to prevent throughput degradation. We describe this algorithm in section 3.4. In this case, the encoder is either encoder-*H* or encoder-*L*. The output data is decided based on the estimated result (#12). In this way, the receiver can determine the importance of the information.

The proposed scheme is a modification of the time multiplexing approach mentioned in previous schemes [3],[4]. However, in the proposed system, no extra information about which code was used is added and the importance level can be distinguished by switching between two codes which use different signal constellations. No extra information results in no decrease in transmission speed of the effective information and no need to consider errors in the extra information.

B. Two RING Signal Constellations

We evaluate a signal constellation, which we call 2RING, that is a combination of two QPSK constellations of different energy [9]. In Fig.2, the 2RING constellation is shown. In Fig.2, signal points 0,1,2,3 are for the code-*L*, while points 4,5,6,7 are for the code-*H*.

C. Code Estimation Algorithm

As we mentioned in section **2.A**-step7, the receiver does not know which code was used to encode the data, the code estimation process is a binary decision problem. So,

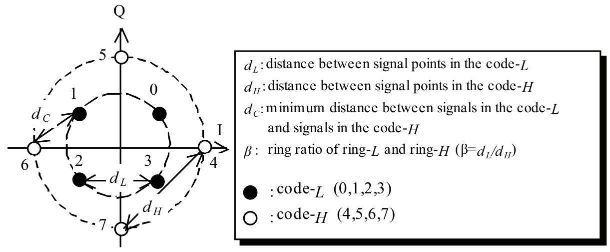

Fig.2 Two RING Signal Constellations (2RING).

the performance of the entire UEP system depends heavily on the performance of the code estimation process. It is our goal to decide which signal sub-constellation the received signal came from using the transmitted signal. We label the i_{th} signal point for code-*L* l_i and the j_{th} signal point for code-*H* h_j. However, $i \in \{0,1,2,3\}$ and $j \in \{4,5,6,7\}$. Also, the l_i, h_j, r_k and n_k are the 2-dimensional vectors and $|\ x\ |$ is the length of the 2-dimensional vector x. In this paper, since we are assuming AWGN channels, the received signal point, r_k, will be either l_i+n_k or h_j+n_k, where n_k is a zero mean Gaussian random variable with variance $N_0/2$. Here, N_0 is the power spectral density of the noise. To decide which code was used, we form decision variables, μ_L and μ_H, for the codes-*L* or *H* respectively as

$$\mu_L = \frac{1}{N}\sum_{k=1}^{N}\left|\min(d_{ki})\right|^2,\ \mu_H = \frac{1}{N}\sum_{k=1}^{N}\left|\min(d_{kj})\right|^2 \tag{1}$$

Our decision rule is to decide that the code-*L* was used If $|\mu_L| < |\mu_H|$ and the code-*H* was used otherwise.

III. THEORETICAL ANALYSES

A. Geometrical Constraint and Average Energy

An analysis model of 2RING is shown and various parameters are defined in Fig.2. Since d_H is larger than d_L, we can write d_L in terms of d_H as

$$d_L = \beta d_H \tag{2}$$

where $0 \leqq \beta < 1$. Now, in order to further parameterize the families so that d_c and d_H are related, we introduce γ such that

$$d_H = \gamma d_c \tag{3}$$

However γ and β cannot be chosen independently, because changing one of the distances results in a change in the two other distances. For 2RING (see Appendix-(A)),

$$\gamma^2 = \frac{2}{\beta^2 - \sqrt{2}\beta + 1} \tag{4}$$

the meaningful range of β is also $0 \leqq \beta < 1$.

Since each transmitted signal has a different energy, the average energy is used to calculate the Signal-to-Noise Ratio (SNR). The average energy is defined as

$$\overline{E} = \frac{1}{K}\sum_{i=1}^{K} E_i \tag{5}$$

where E_i is the energy of the i_{th} signal and K is the number of signals in the constellation. In terms of the signal constellation, E_i is just the squared Euclidean distance of the i_{th} signal point from the origin. The RING family has an average energy given by (see Appendix-

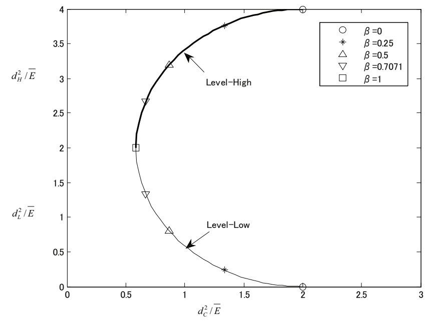

Fig.3 $d_C^2/\overline{E}$ versus $d_H^2/\overline{E}$ (level-H), $d_L^2/\overline{E}$ (level-L)
(The values on the vertical axis: $0 < d_L^2/\overline{E} < 2$, $2 < d_H^2/\overline{E} < 4$).

(A))

$$\overline{E} = \left[\frac{1+\beta^2}{2\left(\beta^2 - \sqrt{2}\beta + 1\right)}\right] d_c^2 \tag{6}$$

B. Code Estimation Error Rate

In section **2.C**, we developed a code estimation algorithm for each signal constellation type using the transmitted signal. In this section, we derive the approximation of the code estimation error rate for the 2RING constellation in AWGN channels. It is given by (see Appendix-(B))

$$P(x > \frac{d_c}{2}) = \sqrt{\frac{\pi - 2}{4\pi N}} erfc\left[\sqrt{\frac{N d_c^2 \overline{E}}{4 N_0}} - \sqrt{\frac{N}{\pi}}\right]. \tag{7}$$

C. Relationship between the signal constellation and the error rate

The error rate performance largely depends on the distances between signal points. Namely, the average bit error rate for each code (codes-H or L) is related to the distance d_H or d_L and the code decision error rate for the 2RING constellation is related to the distance d_c. To evaluate the relationship between the signal constellation and the error rate, we introduce the following minimum squared Euclidean distances normalized by the average energy: $d_H^2/\overline{E}$, $d_L^2/\overline{E}$, $d_C^2/\overline{E}$. They are given by (8),(9),(10).

The average bit error rate for code-H:

$$\frac{d_H^2}{\overline{E}} = -\frac{4\beta^2}{1+\beta^2} + 4 \tag{8}$$

The average bit error rate for code-L:

$$\frac{d_L^2}{\overline{E}} = \frac{4\beta^2}{1+\beta^2} \tag{9}$$

The importance level estimation error rate:

$$\frac{d_C^2}{\overline{E}} = \frac{2(\beta^2 - \sqrt{2}\beta + 1)}{1+\beta^2} \tag{10}$$

In Fig.3, $d_H^2/\overline{E}$ (level-H) and $d_L^2/\overline{E}$ (level-L) are shown versus $d_C^2/\overline{E}$ as a function of β for $0<\beta<1$. The average bit error rate for each code (code-H or L) is related to $d_H^2/\overline{E}$ or $d_L^2/\overline{E}$ and the code decision error rate for the 2RING constellation is related to $d_C^2/\overline{E}$. These error rates decrease as the distances increase. From Fig.3, if we use a smaller β, d_c is larger, so the code decision error rate for the 2RING constellation decreases. Also, d_H is larger and the average bit error rate for the code-H decreases. Similarly, d_L is smaller and the average bit error rate for the code-L increases. If we use a larger β, the opposite is true. Therefore, since the value of β is related to both the average bit error rate for each code and the code decision error rate for the 2RING constellation, a tradeoff exists among them.

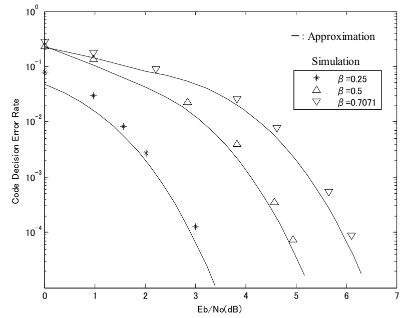

Fig.4 Code Estimation Error Rate versus E_b/N_0
(N=30, β =0.25, 0.5, 0.7071).

IV. PERFORMANCE EVALUATION

We examine the performance of our system by using computer simulations. Moreover, we compare the results with the theoretical analysis described in section **3**. We assume AWGN channels. We use a rate 1/2 code for the code-H and do not use coding for the code-L. Also, the main parameters in the evaluation are the switching rate of the importance level N (30 bits) and the ring ratio β (0.25, 0.5, 0.7071). Here, β is d_L/d_H shown in Fig.2. In Fig.4, the code estimation error rate for the 2RING constellation versus E_b/N_0 is shown. First, regardless of the parameter β the approximation given by (7) and the simulation results described in section **2.C** roughly correspond, so the validity of both was confirmed. One of the causes for the difference is the Gaussian approximation given by (7). Next, we consider the relationship between the ratio of the ring and the error rate. From Fig.4, as the parameter β decreases the code decision error rate for the 2RING constellation is improved, so the validity of the theoretical analysis described in section **3.C** was confirmed. Together with previous work [9], the effectiveness of the proposed system in AWGN channels is shown by both theoretical analysis and computer simulations.

V. CONCLUSIONS

We provide theoretical analyses of the proposed system in relation to the 2RING signal constellation and the TCM codes. The effectiveness of the system for AWGN channels is shown using computer simulations. Our main conclusions are as follows.

(a) We theoretically analyzed the relationship between the ring ratio of the 2RING constellation and the error rate. Since the value of the ring ratio is related to both the average bit error rate for each code and the code decision error rate for the 2RING constellation, a tradeoff exists among them. Also, the validity of the theoretical analysis for the 2RING constellation was confirmed using computer simulations.

(b) The code estimation error rate for the 2RING constellation in AWGN channels was derived. Regardless of the value of the ring ratio, the approximation and the computer simulation results roughly correspond, so the validity of approximate expression was confirmed.

APPENDIX

(A) Calculation of the average energy $\overline{E}$ in each signal constellation and derivations related to β and γ.

For the 2RING type,

$$\overline{E} = \frac{1}{2}\left[\left(\frac{d_L}{\sqrt{2}}\right)^2 + \left(\frac{d_H}{\sqrt{2}}\right)^2\right]. \quad (A.1)$$

Using (2), this equation becomes

$$\overline{E} = \frac{1}{4}\left(1+\beta^2\right)d_H^2. \quad (A.2)$$

In this case, d_c is a function of d_L and d_H as shown by

$$d_c^2 = \left(\frac{d_L}{2}\right)^2 + \left(\frac{d_H}{\sqrt{2}} - \frac{d_L}{2}\right)^2 \quad (A.3)$$

which can be rewritten in terms of d_H by using (2) as

$$d_c^2 = \frac{1}{2}\left(\beta^2 - \sqrt{2}\beta + 1\right)d_H^2 \quad (A.4)$$

then in terms of γ by using (3) as

$$\gamma^2 = \frac{2}{\beta^2 - \sqrt{2}\beta + 1}. \quad (4)$$

Combining this with (A.1) results in

$$\overline{E} = \left[\frac{1+\beta^2}{2\left(\beta^2 - \sqrt{2}\beta + 1\right)}\right]d_c^2. \quad (6)$$

(B) Derivation of approximation of the Code Estimation Error Rate for the 2RING constellation in AWGN channels.

If we assume that point 2 (in the code-L) is transmitted, μ_L and μ_H will be in Fig.2.

$$\mu_L = \frac{1}{N}\sum_{k=1}^{N} n_k^2 \quad (B.1)$$

$$\mu_H = \frac{1}{N}\sum_{k=1}^{N}\left(d_c - |n_k|\right)^2 \quad (B.2)$$

A code estimation error occurs when $\mu_L > \mu_H$. This is equivalent to $\mu_L - \mu_H > 0$ or

$$0 < \frac{1}{N}\sum_{k=1}^{N}\left[n_k^2 - \left(d_c - |n_k|\right)^2\right] \quad (B.3)$$

which can be simplified to

$$\frac{d_c}{2} < \frac{1}{N}\sum_{k=1}^{N}|n_k|. \quad (B.4)$$

Since n_k is a zero mean Gaussian random variable, the probability density function of $|n_k|$ is twice the density of a one-sided Gaussian density, i.e.,

$$f_{|n|}(x) = \frac{2}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{x^2}{2\sigma^2}\right], x \geq 0. \quad (B.5)$$

The mean and variance of $|n_k|$ are given by

$$E\left[|n|\right] = \sqrt{\frac{2\sigma^2}{\pi}}, Var\left[|n|\right] = \sigma^2\left[1 - \frac{2}{\pi}\right]. \quad (B.6)$$

Using the Central Limit Theorem, $\Sigma\ |n_k|/N$ can be approximated by a Gaussian random variable with mean equal to E[$|n|$] and variance equal to Var[$|n|$]/N. Using this approximation, the probability of (B.4) is the integral of the tail of the Gaussian density from $d_c/2$. This results in

$$P(x > \frac{d_c}{2}) = \sqrt{\frac{\pi - 2}{4\pi N}}\, erfc\left[\sqrt{\frac{N d_c^2 \overline{E}}{4N_0}} - \sqrt{\frac{N}{\pi}}\right]. \quad (7)$$

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