WP2-A-2

Performance Evaluation of a Decision Feedback Equalizer with an Adaptive Error Prediction Filter

Satoshi Yamazaki * ***
* Department of Mathematics and System Development Graduate School,
Shinshu University, Nagano-shi Nagano 380-8553
*** Department of Etching System Software Engineering
Tokyo Electron Co., Ltd, Minato-ku Tokyo 107-6325
E-mail: t07a688@shinshu-u.ac.jp

David K.Asano **
** Department of Information Engineering,
Shinshu University, Nagano-shi Nagano 380-8553
E-mail: david@cs.shinshu-u.ac.jp

Abstract— **We proposed an Adaptive Error Prediction Filter with a Decision Feedback Equalizer (AEPF-DFE) to achieve faster convergence and lower computational cost during the tracking period. An AEPF is a linear predictive filter that updates its coefficients using an adaptive algorithm. We got the basic characteristic about an AEPF-DFE and the effectiveness was shown using computer simulations. Specially, using a step size of 0.05 and a forgetting factor of 0.7 for the channels considered in this evaluation, we confirmed the effectiveness and domination of the AEPF-DFE in terms of convergence, bit error rate and computational load by computer simulations.**

I. INTRODUCTION

In recent years the demand for wireless digital communication systems such as mobile phones has increased significantly. However, in wireless communication systems, Inter-Symbol Interference (ISI) due to the effects of multi-path fading occurs, resulting in a performance loss. Techniques to counteract ISI include diversity, adaptive equalization and adaptive array antennas, and so on. Here, we pay attention to an adaptive equalizer as a counter-measure for multi-path fading. Other methods such as diversity and array antennae require extra hardware, but an adaptive equalizer can be implemented using digital signal processing and can therefore be used in mobile devices. Also, the frequency utilization efficiency is not affected by measures used only in the receiver [1]. The main non-linear equalizers that have been studied for use in quickly changing mobile environments are the Decision Feedback Equalizer (DFE) and the Maximum Likelihood Sequence Estimation (MLSE) Equalizer. Fundamentally, MLSE equalizers are better than DFEs in performance. However, especially when the number of delayed waves or the number of modulation levels is large, the computational cost of DFEs is less than that of MLSE equalizers. This makes DFEs more suitable for systems with limited hardware resources and therefore we adopted a DFE for the proposed system [1].

In general, commonly used algorithms to adjust the tap gains in adaptive equalizers to minimize the mean square error are the LMS algorithm and the Recursive Least Square (RLS) algorithm. The LMS algorithm has slow convergence, but the required computation is on the order of the filter length *N*. On the other hand, the RLS algorithm has fast convergence but has an N^2 computational cost. Therefore, there are two approaches: use the low computational cost LMS algorithm and try to improve the convergence speed or use the fast convergence RLS algorithm and try to improve the computational cost. We focus on the former approach and propose an Adaptive Error Prediction Filter (AEPF) with a DFE in cascade called AEPF-DFE. An AEPF is a linear predictive filter that updates its coefficients using an adaptive algorithm. Previously the validity of this approach using a 1st order error prediction filter with an LMS-FIR equalizer [2] and with a blind equalizer [3] has been shown. However, these papers mainly discuss convergence properties, but do not show results for Bit Error Rate (BER) performance, which is an important measure of performance for information transmission. In addition, an approach using an AEPF in a DFE has not been found in previous research.

The rest of the paper is organized as follows. In section 2, we describe previously proposed DFE systems. In section 3, we introduce our AEPF-DFE. In section 4, the AEPF-DFE is evaluated using computer simulation. Finally, conclusions are presented in section 5.

II. PREVIOUS DFE SCHEME

A DFE has been studied by several authors [4-10]. In Fig.1, a general DFE system which consists of a Feed-Forward (FF) filter and a Feed-Back (FB) filter is shown. We give an outline of the DFE's operation below, whose explanation number corresponds to Fig.1.

1. Equalization part

This part consists of a FF filter and a FB filter. When we compare a Linear Equalizer (LE) and a DFE, a DFE operates to reduce ISI by using the FB output data from the decision part through the FB filter. The output of this part is given by (1).

$$y'(n) = \sum_{i=0}^{M} c_i u(n-i) - \sum_{i=M+1}^{M+L} c_i y_d(n-i+M) \qquad (1)$$

2. Decision part

We decide the output of the equalization part and estimate the transmitted symbols. As in the proposed system, when we use QPSK modulation, decisions are made using (2).

$$y_d(n) = \mathrm{sgn}[\mathrm{Re}(y'(nT_s))] + j\,\mathrm{sgn}[\mathrm{Im}(y'(nT_s))]$$
$$\mathrm{sgn}(x) = \begin{cases} 1(x \ge 0) \\ -1(x < 0) \end{cases} \qquad (2)$$

3. Error estimation part

The error e(t) between the ideal equalizer output r(t) and the actual output y'(t) is calculated using (3).

$$e(t) = r(t) - y'(t) \qquad (3)$$

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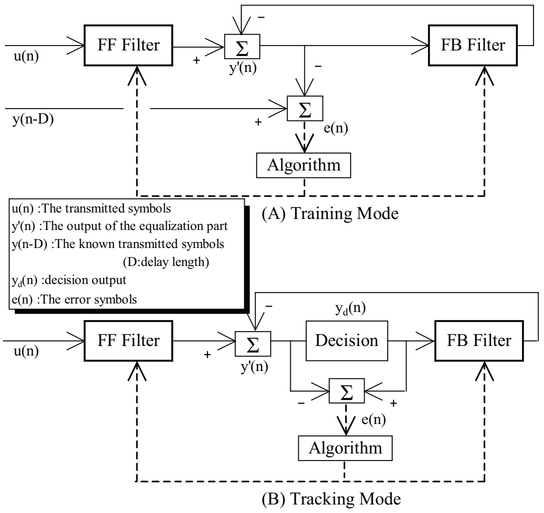

Fig.1 The previous DFE scheme.

Here, the ideal value r(t) is given below.

3.1 training mode: known transmitted symbols y(n - D) (D:a delay)

3.2 tracking mode: decision output $y_d(n)$

4. Tap gain control part

We update the coefficients of both the FF filter and the FB filter with the RLS Algorithm. The RLS algorithm adjusts the tap gains to minimize the mean square error at each tap update. Therefore, it is possible for the equalizer to track in higher speed systems.

III. PROPOSED DFE SCHEME

A. Proposal method

Hereafter, we describe the operation of the proposed scheme. During the training period, we use the RLS algorithm which has a higher convergence speed to focus on transmission efficiency. Thus the "Algorithm" block shown in Fig.1-(A) is the RLS algorithm. On the other hand, during the tracking period the LMS algorithm, which is not computationally intensive, is used to reduce the computational load. However, as the LMS algorithm has a slower convergence speed, we propose a decision feedback equalizer with a linear predictive filter to improve convergence properties. A linear predictive filter consists of a Finite Impulse Response (FIR) filter of order *P*, and future sample values can be predicted from previous values sampled in a fixed time period [11]. The predictive error is the difference between the predicted sample value and the actual sample value. By minimizing the mean square of the predictive errors adaptively using the LMS algorithm, a linear predictive filter has the effect of flattening the frequency response and we can expect an improvement in the convergence speed.

Therefore, we propose an Adaptive Error Prediction Filter (AEPF) with a DFE in cascade called AEPF-DFE. The proposed DFE system is shown in Fig.2 and the structure of the proposed AEPF is shown in Fig.3. Using the proposed method, we expect faster convergence and lower computational cost during tracking. So, when the system is implemented in hardware, we can expect an advantage from the viewpoint of control.

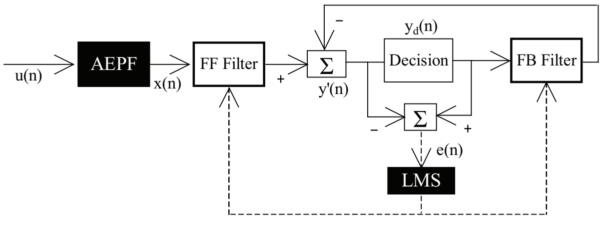

Fig.2 The proposed DFE scheme (AEPF-DFE).

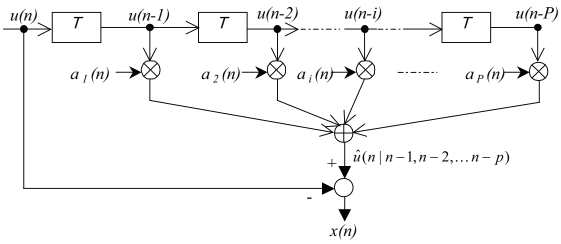

Fig.3 The structure of the proposed AEPF.

B. Formulation

As the AEPF is a linear FIR filter of order *p*, the value predicted from p previous input values is given by (4).

$$\hat{u}(n \mid n-1, n-2, \ldots n-p) = \sum_{i=1}^{p} a_i(n)u(n-i) \tag{4}$$

The output of AEPF, that is, the predictive errors, $x(n)$ is the difference between the input $u(n)$ and the prediction expressed by (4) and is given by (5).

$$x(n) = u(n) - \sum_{i=1}^{p} a_i(n)u(n-i) \tag{5}$$

In other words, we expect the AEPF has the effect of flattening the frequency response by the amount given by expression (4). The prediction error $x(n)$ is the input to the DFE in Fig.2. The tap gains of the AEPF $a_i(n)$ are updated adaptively using the LMS algorithm to minimize the mean square of the prediction errors $x(n)$, $| x(n) |^2$. The vector of tap gains of the AEPF, $\mathbf{a}(n)$, are updated by (6) , whose the initial are zero vector. The vector $\mathbf{u}(n)$ is the vector of the tap inputs.

$$\mathbf{a}(n+1) = \mathbf{a}(n) + \mu x(n)\mathbf{u}(n) \tag{6}$$

Here, μ is the step size parameter.

IV. PERFORMANCE EVALUATION

We evaluate the performance of the AEPF-DFE alone using computer simulations. We compare the previous scheme and the proposed scheme from a point of view of convergence properties, BER properties and the computational load.

A. Simulation Conditions

We compare the previous scheme and the proposed scheme from a point of view of both convergence properties and BER properties. Moreover, we describe the condition under which the proposed scheme becomes more dominant than the previous scheme. The simulation model is shown in Fig.4 and the simulation parameters are shown in Table1. $d(n)$ is a pseudo-random signal, which corresponds to the input signal from the

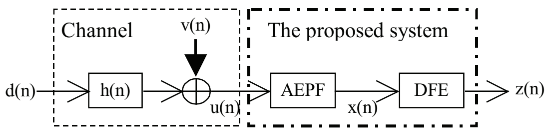

Fig.4 Simulation model.

information source. The input signal to the AEPF is given by (7).

$$u(n) = \sum_{i=0}^{L} h(i)d(n-i) + v(n) \tag{7}$$

Here, $h(n)$ is the impulse response of the channel and $v(n)$ is AWGN. Also, the transfer function of the channel $H(z)$ is the Z-transform of $h(n)$. We assume a baseband transmission communication method. The following two kinds of channel models are considered [2], because in actual wireless communications these two patterns are repeated.

1. Minimum Phase (MP) channel model

$$H(z) = 0.6082 + 0.7603z^{-1} + 0.2280z^{-2} \tag{8}$$

2. Non-MP (NMP) channel model

$$h(n) = \begin{cases} \frac{1}{2}\{1 + \cos(2\pi(n-2)/W)\}, n = 1,2,3 \\ 0, otherwise \end{cases} \tag{9}$$

Equation (9) is called a Raised Cosine channel model [2]. The inter-symbol interference component is controlled by the parameter W. We use W=3.5 in our performance evaluation. Using (5), the input of the AEPF $u(n)$ becomes the input of the DFE $x(n)$. The output of the DFE $z(n)$ becomes the final output signal. Also, we evaluate the following three combinations.

(A) The previous DFE scheme using the LMS algorithm during both training and tracking periods
(B) The previous DFE scheme using RLS during both training and tracking periods
(C) The proposed scheme using RLS during the training period and the LMS algorithm during the tracking period

B. Simulation Results

B.1. Performance of the AEPF-DFE alone

We evaluate the performance of the AEPF-DFE alone. In Fig.5, the dependency on the step-size μ of the LMS algorithm in a MP channel is shown. Here, the step-sizes μ=0.002, 0.01, and 0.05 were used. The following previously reported facts were confirmed for the proposed system. When μ is large, the convergence is fast, but the residual error is large. In contrast, when μ is small, the convergence is slow, but the residual error is small and has stable convergence. In a NMP channel, a delay D for training signals is needed [2]. In Fig.6, the dependency on the delay D is shown. We confirmed that the convergence performance improved as the amount of the delay was increased. Moreover, comparing D=4 to D=8, we see that the amount of improvement is small, so we see that we only have to need a delay of about D=8. In Fig.7, the dependency on the order of the AEPF filter *p* is shown. Here, the order of the AEPF filter used was *p* = 20, 10, 6 and 2. In general, if the order of the filter is small, the amount of processing can be made small. Therefore, minimization of the MSE can be achieved quickly. In Fig.7, we can see that we only need to use a single digit filter order for the AEPF filter.

B.2. Comparision the previous scheme and the proposed scheme

First, in Fig.8, the convergence properties in a MP channel are shown. In Fig.9, convergence properties in a NMP channel are shown. In both figures, the horizontal axis is the number of equalization iterations and the vertical axis is the Mean Square Error (MSE) of the equalization output. In both MP and NMP channels, the effectiveness of the proposed scheme is shown by comparing (A) and (C). In general, the convergence using the LMS algorithm depends on the step size μ and the convergence using the RLS algorithm depends on the forgetting factor λ. The parameters μ, λ are determined according on a case by case basis. If we use values of approximately μ=0.05 and λ=0.7, the proposed scheme (C) has better performance than the previous scheme (B) as can be seen by comparing (B) and (C) in terms of minimum MSE after about 200 iterations.

Second, in Fig.10, the BER properties in an MP channel are shown. In Fig.10, the BER properties in an NMP channel are shown. From both Fig.9 and Fig.11, we can see that comments similar to the ones for the convergence properties can be made about the BER properties. The reason for this is that better convergence leads to a smaller MSE, which results in reduced interference due to multipath and other effects.

Finally, we evaluate the computational load during the tracking periods for (B) and (C). In general, if the filter length is N, the computational cost of the LMS algorithm is $2N+1$ and that of the RLS algorithm is $2.5N^2+4.5N$ [2]. For (B), as the RLS algorithm is used in both FF and FB filters, the computational load is $5N^2+9N$. On the other hand, for (C) as the LMS algorithm is used in AEPF, FF and FB filters, the computational load is $6N+3$. Therefore, we confirmed the effectiveness and domination of the proposed AEPF-DFE in terms of convergence, BER and computational load.

V. CONCLUSIONS

We proposed an Adaptive Error Prediction Filter with a Decision Feedback Equalizer (AEPF-DFE) to achieve faster convergence and lower computational cost during the tracking period. We got the basic characteristic about an AEPF-DFE and the effectiveness was shown using computer simulations. Specially, using a step size of 0.05 and a forgetting factor of 0.7 for the channels considered in this evaluation, we confirmed the effectiveness and domination of the AEPF-DFE in terms of convergence, Bit Error Rate and computational load by computer simulations.

Now, we are currently evaluating the performance when the various parameters are changed and in selective fading channels for actual wireless communications. Moreover, we are estimating theoretical analyses of the proposed system.

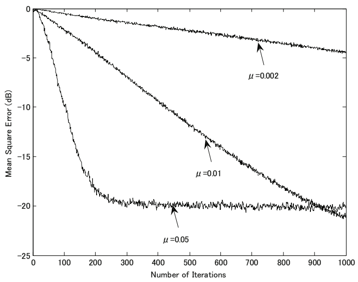

Fig.5 The dependency on the step-size μ in MP channels.

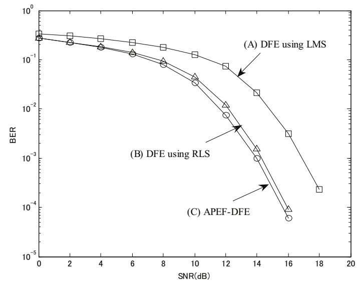

Fig.10 Bit error rate properties in MP channels.

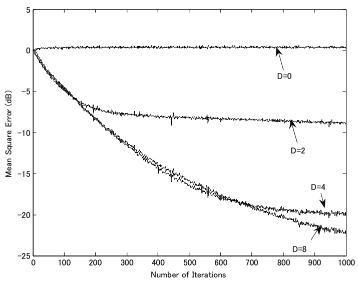

Fig.6 The dependency on the delay D in NMP channels.

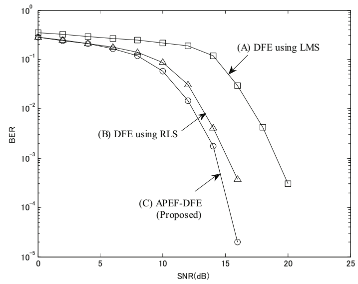

Fig.11 Bit error rate properties in NMP channels.

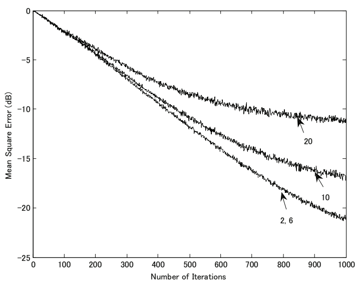

Fig.7 The dependency on the order of the AEPF filter *p* in NMP channels.

Table1 Simulation parameters.

Channel	MP, NMP, AWGN
Raised cosine parameter W	3.5
SNR (Convergence properties)	30[dB]
Number of iterations	1000
Delay of training sequence D	8
Order of APEF filter	8
Order of FF filter	8
Order of FB filter	3
Frame structure	
Training part	200 symbols
Information part	200 symbols
Step size μ on LMS	0.05
Forgetting factor λ on RLS	0.7

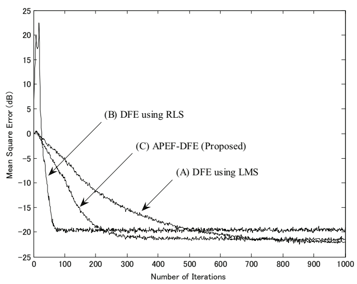

Fig.8 Convergence properties in MP channels.

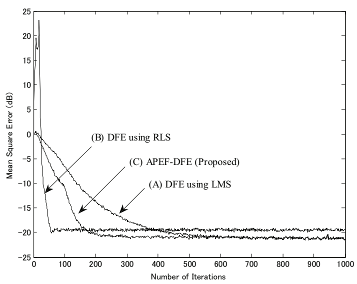

Fig.9 Convergence properties in NMP channels.

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