

FREQUENCY DOUBLE REUSE FOR  
INDOOR AND URBAN DIGITAL  
CELLULAR TELEPHONE SYSTEMS  
– ENHANCED CONCEPTUAL DESIGN  
FORMULAE FOR SINGLE HANDSET  
SYSTEMS –

Yasuaki Kinoshita  
and David K. Asano

*Shinshu University  
Department of Information Engineering  
500 Wakasato, Nagano 380  
Japan*

**Abstract:** This year in Japan, the standardization of the digital frequency double reuse system was proposed and the standardization process is proceeding. This paper presents conceptual design formulae for a digital system. These formulae enhance the previous analog design formulae by introducing two parameters in the wave propagation model.

## INTRODUCTION

In 1989, the first author proposed the frequency double reuse (DR) technique, which allows urban frequency channels to be also used in indoor wireless PBX systems [1] [2]. This technique makes it more convenient to be able to access both urban and indoor systems with a single handset. Moreover, compared to conventional systems, which use adjacent frequency bands, and dual band systems, which have two sets of wireless circuits built into a single handset, it is possible to design a system with higher spectral efficiency. Analog and digital frequency DR systems have already been

standardized (IS-94 and IS-136) and are in use in the United States [7]. In Japan, NTT Docomo proposed the standardization of a digital system, known as PDC (Personal Digital Cellular), and the standardization process has been begun by the Association of Radio Industry Businesses (ARIB).

In this paper, we consider propagation in suburban environments which consist of low-rise concrete buildings. In this type of building, the propagation between floors is heavily attenuated, so the cells on one floor can be considered to be approximately independent of the cells on another floor. Therefore, a simplified two-dimensional propagation model is used in our analysis.

In previously reported results on the conceptual design of frequency DR systems [7], cluster sizes of 10 or more for analog systems were considered. However, for digital systems with a cluster size of less than 10, an improved model is necessary.

In this paper, we present enhanced conceptual design formulae for digital systems. These formulae are derived from the previous formulae by introducing two new parameters in the wave propagation model. These parameters represent the distance after which the propagation of the interference waves from the urban system to the indoor system and vice-versa changes. The enhanced design formulae are more widely applicable because the newly introduced parameters make the propagation model match the DR wireless communication environment more closely.

## FREQUENCY DOUBLE REUSE

The concept of frequency DR is shown in Figure 1. The large “A” represents a macrocell where the frequency **A** is used in an urban cellular system. The small “A” represents a picocell where the frequency **A** is used with small power in an indoor cellular system. In the conventional cellular system, the frequency “A” is reused at a certain distance from the present cell. However, inside the thick lines in the figure, which define clusters, different frequencies, e.g., **B**, **C**, are used. The frequencies are reused in adjacent clusters. In the proposed system, these frequencies are reused again inside the clusters for an indoor cellular system, so this scheme was named frequency double reuse (DR).

The first design parameter in a DR system is the number of “white” cells, i.e., the number of macrocells which contain no picocells in each cluster. These white cells provide the urban system protection from the indoor cellular system. Inside these white cells, frequency **A** is not reused, but other frequencies can be reused. We denote the number of guard cells  $C_g$ . Since the guard cells are placed evenly around the macrocell antenna, the number of guard cells can be represented in terms of the number of guard layers,  $s$  as

$$C_g = 1 + \sum_{k=0}^s k. \quad (1)$$

Therefore, in Figure 1,  $s = 0$  and  $C_g = 1$ .

The number of frequency channels used in the urban cellular system is denoted by  $C_1$  and referred to as the cluster size. In Figure 1,  $C_1 = 13$ .

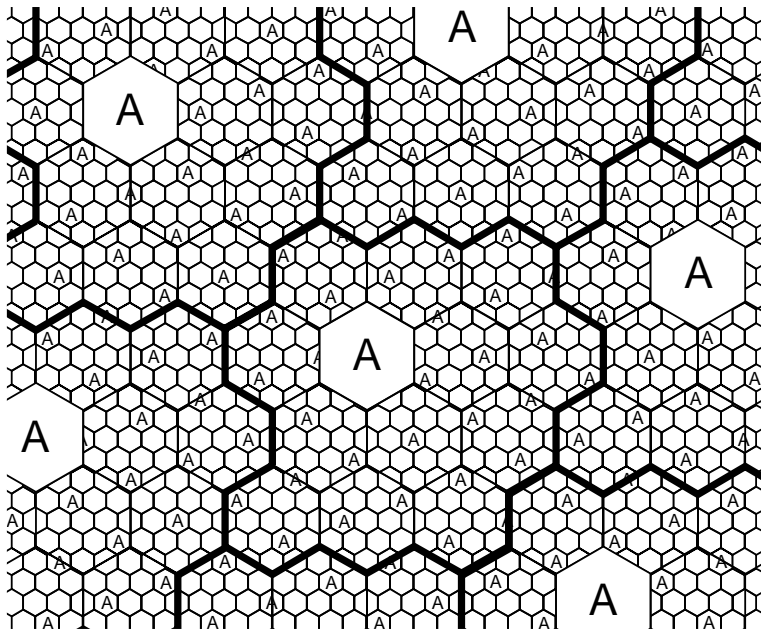


Figure 1 A typical frequency double reuse system. The large “A”s show the macrocells where the frequency **A** is used. The small “A”s show the picocells where the frequency **A** is reused with low power for the indoor cellular system.

The basic design criterion for DR systems is that the signal to interference ratios (SIRs) of the urban and indoor cellular systems that use the same frequency are the same. In mathematical terms, this condition can be written as

$$\frac{S_1}{I_1 + J_{21}} = \frac{S_2}{I_2 + J_{12}}, \quad (2)$$

where  $S_1$  and  $S_2$  are the signal powers at the macrocell and picocell edges, respectively,  $I_1$  and  $I_2$  are the self cochannel interferences for the urban (macrocell) and indoor (picocell) systems, respectively,  $J_{21}$  is the mutual cochannel interference that the indoor system causes on the urban system and  $J_{12}$  is the mutual cochannel interference that the urban system causes on the indoor system. This criterion means that the service quality is the same for both systems. The SIRs for the picocell and macrocell systems are worst at the edges of the cells, so the design equations are derived when (2) is satisfied at the cell edges.

In this paper, the subscript “1” refers to the urban system, while the subscript “2” refers to the indoor system. The subscripts  $i$  and  $j$  that appear later can take a value of 1 or 2 to refer to the urban or indoor system, respectively.

The next design parameters are  $\beta_1$  and  $\beta_2$ , which are defined as

$$\beta_1 = \frac{J_{21}}{I_1} \text{ and } \beta_2 = \frac{J_{12}}{I_2}. \quad (3)$$

These parameters are decided when the transmission power of the indoor cellular system and the picocell radius is designed. This will be explained later.

## ENHANCED DESIGN PROPAGATION MODEL

It is well known that the mean signal power at a distance  $r$  from an antenna in a fading environment is given by

$$S_i = K_i \frac{P_i}{r^{\alpha_i}}, \quad (4)$$

where  $P_i$  is the transmitter power,  $\alpha_i$  is an attenuation parameter and  $K_i$  is a constant (Appendix 2). The self and mutual cochannel interferences are calculated by adding the effects of many base station antennas using the same frequency.

In Figure 2, the propagation model that is used to calculate the mutual interferences in this paper is shown compared to model previous model [1][2] [7]. The signal strength,  $S$ , is plotted as a function of the distance from the base station antenna,  $r$ . The signal strength decreases with a propagation exponent of  $\alpha = 2$  from the base station antenna for the line-of-sight distance  $\Lambda_i$ . After that, it is well known that the signal strength decreases with  $\alpha_1 = 3.5$  for urban propagation, [4] and  $\alpha_2 = 4.5$  for indoor propagation in a typical steel reinforced concrete building [5].

In this paper, two new parameters,  $\rho_1$  and  $\rho_2$  are introduced. The parameter  $\rho_1$  is the distance after which the propagation of the interfering wave from the urban system to the indoor system changes, while  $\rho_2$  is the corresponding distance for the interfering wave from the indoor system to the urban system. However, the propagation of the waves involved in cochannel interference, i.e., in the calculation of  $I_i$ , does not change.

In (2), if the two systems do not affect each other, i.e.,  $J_{12} = J_{21} = 0$ , then the communication quality of the two systems is determined by only the ratio of the signal strength and cochannel interference  $S/I$ . In this case, from the criterion that the communication quality of the two systems is the same, i.e.,  $S_1/I_1 = S_2/I_2$ , the cluster size of the indoor cellular system,  $C_2$ , can be calculated from the urban system's cluster size using [1] [2]

$$C_2 = 0.71418C_1^{7/9}. \quad (5)$$

Since the attenuation in the indoor system is greater than that in the urban system, the indoor system can achieve the same communication quality with a smaller cluster size.

Next, in order to use the same frequency channel in both systems, we rewrite (2) for the case when  $\beta_1 = \beta_2 = \beta$  as

$$\frac{J_{21}}{S_1} = \frac{J_{12}}{S_2}. \quad (6)$$

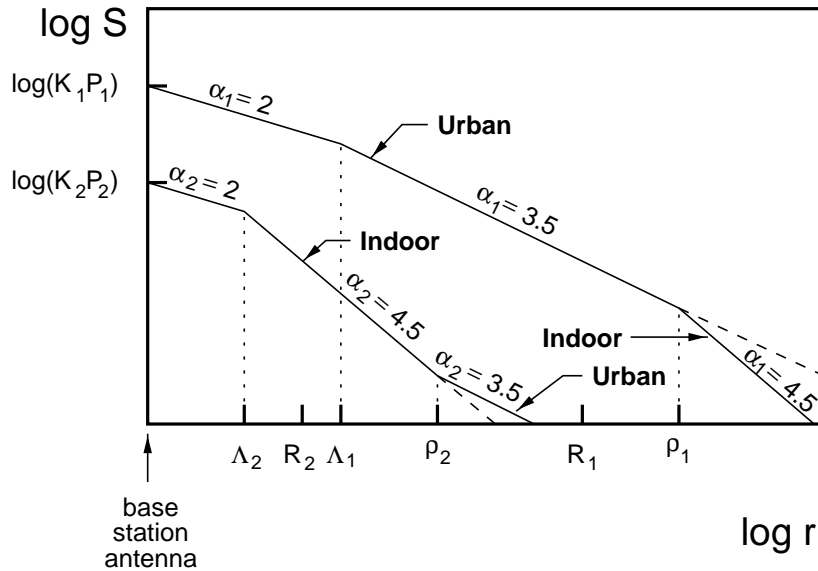


Figure 2 The signal propagation model used in this paper. The parameters  $\rho_1$  and  $\rho_2$  are introduced to take into account the change in propagation from an indoor environment to an urban environment and vice-versa. The model used in previous papers is shown with a dashed line.

In this paper, we imagine the situation when there is an infinite number of evenly distributed cells in the indoor system. The solution of (6) gives us the conceptual design formula for a DR system.

**INDOOR SYSTEM CONCEPTUAL DESIGN FORMULA**

**Power Ratio**

The calculation of the self and mutual cochannel interferences are shown in Appendix 2. Using these results, we can derive the following conceptual design formula from (6): (Appendix 2)

$$\frac{P_2}{P_1} = \left(\frac{R_2}{R_1}\right)^{\frac{\alpha_2}{2}+1} \left(\frac{\sqrt{\rho_1\rho_2}}{\Lambda_2}\right)^{\alpha_2-\alpha_1} \left(\frac{\Lambda_1}{\Lambda_2}\right)^{\alpha_1-2} E(C_2, C_g), \quad (7)$$

where  $E$  is a function that depends only on the structure of the cell model and the propagation constants  $\alpha_i$ . If this function is calculated once and tabulated, it is possible to perform a conceptual design easily.

### Frequency Utilization Efficiency Calculation

The frequency utilization efficiency per unit area,  $\gamma$ , is defined by

$$\frac{\text{phone traffic[erlangs]}}{\text{bandwidth[MHz]} \cdot \text{area[km}^2\text{]}}. \quad (8)$$

For DR systems, the frequency utilization efficiency relative to that of conventional systems,  $\gamma_r$ , is given by

$$\gamma_r = (1 + \beta)^{\frac{-2}{\alpha_1}} + (1 + \beta)^{\frac{-2}{\alpha_2}} \left(1 - \frac{C_g}{C_1}\right), \quad (9)$$

where  $\beta$  is given by (Appendix 2)

$$\beta = \left(\frac{R_2}{R_1}\right)^{\frac{\alpha_2}{2}-1} \left(\frac{\rho_1}{\rho_2}\right)^{\frac{\alpha_2-\alpha_1}{2}} H(C_2, C_g). \quad (10)$$

Here,  $H$  is a function that only depends on the structure of the cell model and  $\alpha_i$ . Similarly to  $E$ , if this function is tabulated, a conceptual design can be easily done.

### NUMERICAL RESULTS

The parameters introduced in this paper,  $\rho_1$  and  $\rho_2$  are strictly speaking a function of the picocell coordinates. However, in order to be able to get some results in this analysis, we assume the parameters are constants. As a result, these parameters appear as a product in (7) and a ratio in (10).

In digital cellular systems, the cluster size is smaller than in analog systems. In practice, 3-sector cells are used in cellular phone systems. The number of sectors can be thought of as approximately the same as the cluster size.

The values of  $E$  and  $H$  were calculated and the results are shown in Table 1. For comparison, the values when  $\alpha_1 = \alpha_2 = 3.5$  are also shown.

In order to complete a conceptual design, the parameters shown in the propagation model in Figure 2,  $\Lambda_i$  and  $\rho_i$ , must be decided. To keep the estimate of the mutual interference conservative, we assume  $\rho_1 = 2R_1$  and  $\rho_2 = 0.5R_2$ . This results in values of  $\beta$  twice as large as when  $\rho_1 = R_1$  and  $\rho_2 = R_2$ . From previous work [1] [2], we use  $\Lambda_1 = 120m$  and  $\Lambda_2 = 3m$ . Using these values, numerical results, which fit the environment at our university, are shown in Table 2.

An indoor cellular system with a base station transmission power of  $P_2 = 10mW$  has a cell radius of approximately  $R_2 = 30m$ . Therefore, even for digital systems with macrocell cluster sizes less than 10, it is possible to obtain service area values that are close to the values obtained from experience [1][2][7].

The parameters,  $\rho_1$  and  $\rho_2$  are roughly the same order as the cell radii of the two systems. For macrocell cluster sizes of 7 or less, it is necessary to rewrite the conceptual design formula when sector cells are used. For three sector urban systems, approximate solutions can be obtained by using  $3C_1$  instead of  $C_1$  in this analysis.

Table 1  $E(C_2, C_g)$  and  $H(C_2, C_g)$

$C_g$	$C_1$	$E$	$H$	$E$	$H$
		$(\alpha_1 = \alpha_2 = 3.5)$		$(\alpha_1 = 3.5, \alpha_2 = 4.5)$	
1	3	1.23	2.44	0.654	0.843
	4	1.16	3.06	0.590	0.963
	7	1.08	4.63	0.511	1.230
	9	1.14	6.01	0.539	1.523
	12	1.26	8.27	0.586	1.979
	13	1.30	9.07	0.602	2.134
	16	1.41	11.6	0.647	2.601
	19	1.52	14.2	0.689	3.067
	21	1.59	16.1	0.715	3.379
	25	1.72	19.9	0.763	4.000
7	12	1.47	2.03	0.476	0.338
	13	1.47	2.32	0.487	0.388
	16	1.30	2.72	0.403	0.411
	19	1.18	2.98	0.343	0.414
	21	1.17	3.29	0.340	0.447
	25	1.16	3.86	0.334	0.505

Table 2 Microcell Radius  $R_2$  and Frequency Utilization Efficiency  $\gamma_r$

$C_1$	$C_2$	$C_g$	$E(C_2, C_g)$	$H(C_2, C_g)$	$R_2$	$\gamma_r$	$\beta$
7	3.2	1	0.511	1.23	28.4 m	185%	0.0073
9	3.9	1	0.539	1.52	28.0 m	188%	0.0088
16	6.2	1	0.647	2.60	26.7 m	192%	0.0142
16	6.2	7	0.403	0.41	30.3 m	156%	0.0026

$$P_1 = 3W, R_1 = 3km, P_2 = 10mW, \rho_1 = 2R_1, \rho_2 = 0.5R_2$$

The frequency utilization efficiency is close to the theoretical limit of 200%. The reason for this is that the urban cellular system and the indoor system can be thought to be designed to have a very high degree of electrical screening.

To examine the accuracy of this analysis, we compared the results using the new design formulae with those using the previous formulae. In order to obtain approximately the same indoor parameters  $R_2$  and  $P_2$  for the given urban parameters  $R_1$  and  $P_1$ , we have to decrease  $\Lambda_2$  by one-half to one-third for the same  $\Lambda_1$ . We found that

the newly introduced design parameters  $\rho_1$  and  $\rho_2$  allow us to successfully match the propagation model to complicated urban and indoor environments.

## CONCLUSIONS

In this paper, conceptual design formulae for digital frequency double reuse cellular systems with cluster sizes less than 10 was derived and numerical results shown. This extension was made by introducing two new parameters in the propagation model to improve the calculation of the cochannel interference. These parameters are roughly the same as the cell radii of the urban and indoor systems. For sector cell systems, it is necessary to extend the current formulae with a new cell model. In future work, the analysis of spread spectrum double reuse systems will be done.

### Appendix: Received Signal Strength

The received signal strength,  $S$ , of UHF band signals in urban environments is given by (4) and for a distance,  $r$ , of 1 to 10 km,  $\alpha$  is approximately 3.5 [4]. The constant  $K_i$  in (4) is given by

$$K_i = \left( \frac{\lambda}{4\pi} \right)^2 G_t G_r \Lambda_i^{\alpha_i - 2}, \quad (\text{A.1})$$

where  $\lambda$  is the wavelength of the signal,  $G_t$  and  $G_r$  are the transmitter and receiver antenna gains, respectively, and  $\Lambda_i$  is the distance over which inverse square law propagation applies, i.e., the line of sight distance.

In Figure 2, the signal strength for distances greater than  $\rho_i$  is given by

$$S_i = K_i \rho_i^{\alpha_i - \alpha_j} \frac{P_i}{r^{\alpha_j}}, \quad r > \rho_i. \quad (\text{A.2})$$

### Appendix: Self and Mutual Cochannel Interference

The self cochannel interferences,  $I_1$  and  $I_2$ , are calculated by adding the signal powers from all cells using the same frequency. If these cells are numbered  $\{1, \dots, \infty\}$ , the cochannel interference can be written as

$$I_1 = \sum_{m=1}^{\infty} K_1 \frac{P_1}{r_{1m}^{\alpha_1}} \quad \text{and} \quad I_2 = \sum_{m=1}^{\infty} K_2 \frac{P_2}{r_{2m}^{\alpha_2}}, \quad (\text{B.1})$$

where  $r_{1m}$  is the distance from the  $m$ th interfering cell to the mobile user using the macrocellular system and  $r_{2m}$  is the corresponding distance for the picocellular system. Here, we assume that the propagation exponent for the cochannel interference between macrocells and between picocells does not change after  $\Lambda_2$  and  $\Lambda_1$ , respectively.

Normalizing  $r_{1m}$  and  $r_{2m}$  by the macrocell and picocell radii, respectively, results in

$$I_1 = K_1 \frac{P_1}{R_1^{\alpha_1}} \sum_{m=1}^{\infty} \frac{1}{r_{1m}^{\alpha_1}} \quad (\text{B.2})$$



and

$$I_2 = K_2 \frac{P_2}{R_2^{\alpha_2}} \sum_{m=1}^{\infty} \frac{1}{r_{2m}^{\alpha_2}}, \quad (\text{B.3})$$

where

$$r_{1m} = \frac{r_{1m}}{R_1} \quad \text{and} \quad r_{2m} = \frac{r_{2m}}{R_2} \quad (\text{B.4})$$

The summations above now only depend on the attenuation exponents and the distances between cells of unit radii. We define the summation part of the above equations as  $\bar{I}_1$  and  $\bar{I}_2$ , which results in

$$I_1 = K_1 \frac{P_1}{R_1^{\alpha_1}} \bar{I}_1 \quad \text{and} \quad I_2 = K_2 \frac{P_2}{R_2^{\alpha_2}} \bar{I}_2, \quad (\text{B.5})$$

where

$$\bar{I}_1 = \sum_{m=1}^{\infty} \frac{1}{r_{1m}^{\alpha_1}} \quad \text{and} \quad \bar{I}_2 = \sum_{m=1}^{\infty} \frac{1}{r_{2m}^{\alpha_2}}. \quad (\text{B.6})$$

The mutual interference is the interference that the urban and indoor cellular systems cause on each other. The interference from the urban system on the indoor system, i.e., macrocell on picocell, is denoted by  $J_{12}$ , while picocell on macrocell interference is denoted by  $J_{21}$ .

If we denote the distance from the picocell system user to the  $m$ th macrocell antenna as  $d_{2m}$ , we can write  $J_{12}$  as

$$J_{12} = K_1 \rho_1^{\alpha_2 - \alpha_1} P_1 \sum_{m=1}^{\infty} \frac{1}{d_{2m}^{\alpha_2}}. \quad (\text{B.7})$$

To calculate  $J_{21}$ , we assume that the picocell antennae which are inside a macrocell are located at the center of the macrocell. The average number of interfering picocells in a macrocell can be approximated by

$$l = \frac{1}{C_2} \left( \frac{R_1}{R_2} \right)^2. \quad (\text{B.8})$$

Therefore, if we denote the distance from the macrocell user to the center of the  $m$ th macrocell which contains picocells as  $d_{1m}$ ,  $J_{21}$  can be written as

$$J_{21} = K_2 \rho_2^{\alpha_1 - \alpha_2} P_2 l \sum_{m=1}^{\infty} \frac{1}{d_{1m}^{\alpha_1}}. \quad (\text{B.9})$$

It should be remembered here that there are no picocells inside “white” macrocells.

We normalize the distances by  $R_1$  to get

$$J_{12} = K_1 \rho_1^{\alpha_2 - \alpha_1} \frac{P_1}{R_1^{\alpha_2}} \bar{J}_{12} \quad (\text{B.10})$$

and

$$J_{21} = K_2 \rho_2^{\alpha_1 - \alpha_2} \frac{P_2}{R_1^{\alpha_1}} l \check{J}_{21}, \quad (\text{B.11})$$

where

$$\bar{J}_{12} = \sum_{m=1}^{\infty} \frac{1}{\underline{d}_{2m}^{\alpha_2}} \quad \text{and} \quad \check{J}_{21} = \sum_{m=1}^{\infty} \frac{1}{\underline{d}_{1m}^{\alpha_1}} \quad (\text{B.12})$$

and

$$\underline{d}_{1m} = \frac{d_{1m}}{R_1} \quad \text{and} \quad \underline{d}_{2m} = \frac{d_{2m}}{R_1} \quad (\text{B.13})$$

### Appendix: Power Ratio Derivation

Substituting (4), (B.10) and (B.11) into (6) gives,

$$\frac{P_2}{P_1} \frac{K_2}{K_1} \frac{R_1^{\alpha_1}}{R_2^{\alpha_2}} = \frac{K_1}{K_2} \frac{\rho_1^{\alpha_2 - \alpha_1}}{\rho_2^{\alpha_1 - \alpha_2}} \frac{P_1}{P_2} \frac{C_2 R_2^2}{R_1^2} \frac{R_1^{\alpha_1} \bar{J}_{12}}{R_1^{\alpha_2} \check{J}_{21}}. \quad (\text{C.1})$$

Rearranging terms gives

$$\frac{P_2^2}{P_1^2} = \frac{K_1^2}{K_2^2} (\rho_1 \rho_2)^{\alpha_2 - \alpha_1} \frac{R_2^{\alpha_2 + 2} \bar{J}_{12}}{R_1^{\alpha_2 + 2} \check{J}_{21}} C_2. \quad (\text{C.2})$$

From (A.1),  $K_1/K_2 = \Lambda_1^{\alpha_1 - 2} / \Lambda_2^{\alpha_2 - 2}$ . Substituting this into the above equation and taking the square root gives the desired result:

$$\frac{P_2}{P_1} = \left( \frac{R_2}{R_1} \right)^{\frac{\alpha_2}{2} + 1} \left( \frac{\sqrt{\rho_1 \rho_2}}{\Lambda_2} \right)^{\alpha_2 - \alpha_1} \left( \frac{\Lambda_1}{\Lambda_2} \right)^{\alpha_1 - 2} E(C_2, C_g), \quad (\text{C.3})$$

where

$$E(C_2, C_g) = \sqrt{\frac{\bar{J}_{12}}{\check{J}_{21}}} C_2. \quad (\text{C.4})$$

### Appendix: Interference Ratio Derivation

Substituting (B.10) and (B.5) into (3) gives

$$\beta = \frac{K_1 (\rho_1)^{\alpha_2 - \alpha_1} \frac{P_1}{R_1^{\alpha_2}} \bar{J}_{12}}{K_2 \frac{P_2}{R_2^{\alpha_2}} \bar{I}_2}. \quad (\text{D.1})$$

Rearranging terms gives

$$\beta = \frac{K_1 P_1}{K_2 P_2} \rho_1^{\alpha_2 - \alpha_1} \left( \frac{R_2}{R_1} \right)^{\alpha_2} \frac{\bar{J}_{12}}{\bar{I}_2}. \quad (\text{D.2})$$

Using (C.2) to replace  $K_1 P_1 / (K_2 P_2)$  results in the desired result:

$$\beta = \left(\frac{R_2}{R_1}\right)^{\frac{\alpha_2}{2}-1} \left(\frac{\rho_1}{\rho_2}\right)^{\frac{\alpha_2-\alpha_1}{2}} H(C_2, C_g), \quad (\text{D.3})$$

where

$$H(C_2, C_g) = \frac{1}{I_2} \sqrt{\frac{\tilde{J}_{21} \bar{J}_{12}}{C_2}}. \quad (\text{D.4})$$

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