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EFFICIENT BLOCK AND CONVOLUTIONAL CODING OF GMSK

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Abstract –In this paper, block and convolutional coding for Gaussian Minimum Shift Keying is investigated. The improvement in performance that can be obtained without an increase in bandwidth is found as a function of the block length for block codes and as a function of the constraint length for convolutional codes. For a fixed system transmission bandwidth, the tradeoff between the percentage of bandwidth to use for coding and the percentage to use for modulation is examined and optimized for various values of bandwidth.

I. Introduction

Gaussian Minimum Shift Keying (GMSK) is a popular modulation method that was first proposed in 1981 [1]. It has since been adopted as the modulation scheme for the GSM digital cellular system due to such favourable properties as bandwidth efficiency and constant envelope. Although the performance of GMSK has been analyzed by several researchers, e.g., [2][3][4][5][6], optimization of coding for GMSK has not been examined.

It is well known that coding increases the required transmission bandwidth when considered independently from data modulation. However, what the best tradeoff between modulation and coding is for a given transmission bandwidth is not well known. In this paper, we examine this tradeoff for coded GMSK systems and determine the optimal amount of coding for a range of transmission bandwidths. We also show the amount of gain that can be achieved compared to an uncoded system that uses the same transmission bandwidth.

In order to keep the bandwidth of the coded system the same as the uncoded system, we use the coding method shown in Fig. 1. As shown, the encoder in the coded system increases the number of bits that must be transmitted. Therefore, the bit interval of the coded bits must be shorter than the bit interval of the uncoded bits in order to keep the information transmission rate constant.

A shorter bit interval results in a larger transmission bandwidth. Therefore, to compensate for this in the coded system, the GMSK modulator used produces a narrower spectrum compared to the modulator used in the uncoded system. Thus, the uncoded and coded systems with the same bandwidth can be produced.

The rest of the paper is organized as follows. First, we start with some preliminary discussion of GMSK and the performance measures necessary to evaluate the system. Second, we examine the coding and modulation tradeoffs for block codes. Third, we examine the tradeoffs for convolutional codes and then finish with some conclusions.

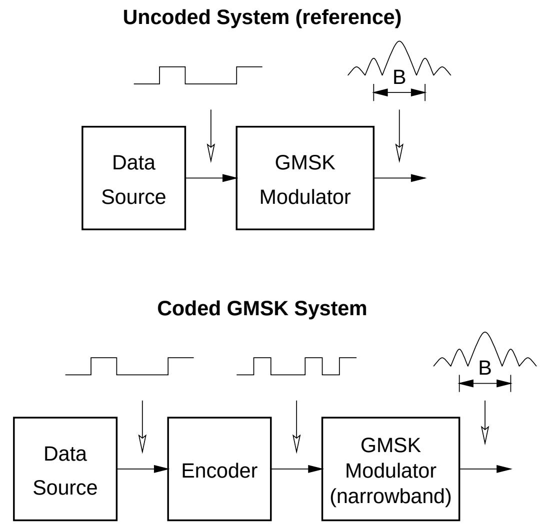

Fig. 1. The coded GMSK system examined in this paper compared to the uncoded system used as a reference.

II. Preliminaries

A. GMSK Modulation

A GMSK signal, in the kth bit interval, can be expressed as

$$s(t) = A\cos\left(2\pi f_c t + \pi \sum_k a_k \int_{kT}^{t} g(\tau - kT)d\tau\right), \tag{1}$$

where A is the signal amplitude, a_k is the input data, T is the bit interval and $g(t)$ is the frequency pulse, which determines the overall spectral characteristics of the modulated signal. The bit interval for the coded system, T_c, is related to that for the uncoded system, T, by

$$T_c = rT, \tag{2}$$

where r is the rate of the code.

For GMSK, $g(t)$ is given by [4]

$$g(t) = K\{\mathrm{erf}[cB_b(t+\frac{T}{2})] - \mathrm{erf}[cB_b(t-\frac{T}{2})]\}, \tag{3}$$

where K is a constant chosen so that the pulse area is equal to $1/2$, B_b is the $3dB$ bandwidth of the Gaussian prefilter used in GMSK modulators and c is a constant given by $c = \pi\sqrt{2/\ln 2}$. The frequency pulse is a continuous function that is truncated so that it is zero outside the interval $[-LT/2, LT/2]$. The parameter L is the memory length of the modulation, i.e., the number of symbols which a single symbol can affect. In this paper, we use $L = 3$. The spectrum of a GMSK signal, and hence the bandwidth, can be easily controlled by the parameter B_b. A larger value of B_b results in a larger signal bandwidth.

B. Bandwidth

In order to determine what combination of coding rate and B_b for the coded system results in the same bandwidth as the uncoded system, we must choose a measure of bandwidth. In this paper, we use the percent power containment bandwidth, denoted by B_x and defined as the bandwidth which contains $x\%$ of the signal power, e.g., $B_{99.9}$ is the bandwidth that contains 99.9% of the signal power. In this paper, we use $B_{99.9}$, since this includes the effects of side lobes.

C. Error Rate

For transmission in an AWGN (Additive White Gaussian Noise) channel, the bit error rate performance of GMSK at high SNR (signal-to-noise ratio) values is given by

$$p = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{d_{min}^2 E}{2N_0}}\right), \tag{4}$$

where d_{min} is the normalized minimum Euclidean distance between the signal representing '0' and the signal representing '1', E is the energy of the transmitted signal and $N_0/2$ is the power spectral density of the additive white Gaussian noise, i.e., the SNR is equal to E/N_0.

The normalized minimum Euclidean distance for GMSK is defined and plotted in [1]. As B_b is increased, d_{min}^2 approaches a maximum value of 2, which corresponds to the performance of optimal antipodal signalling.

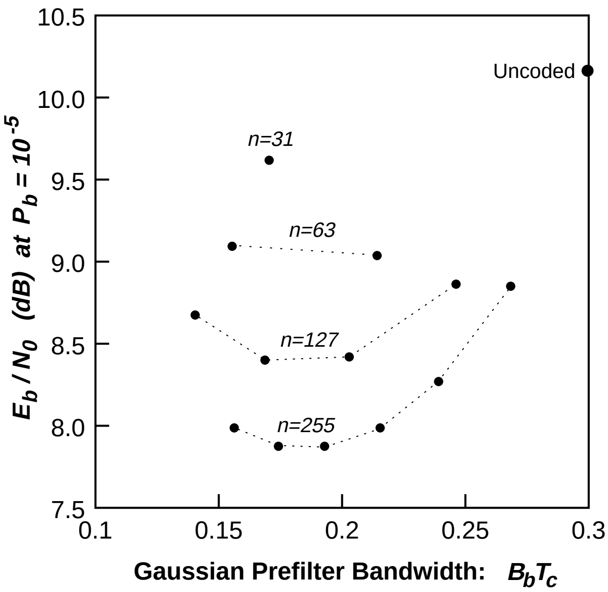

Fig. 2. The SNR required to achieve a bit error rate of 10^{-5} as a function of the normalized Gaussian prefilter bandwidth, B_bT_c, for various BCH codes with block length n. The total system bandwidth is equal to $B_{99.9}T = 1.167$, which corresponds to an uncoded system with $B_bT = 0.3$.

To make a fair comparison between the uncoded and coded systems, the value of E used in (4) for the uncoded system is E_b, which is the energy per transmitted bit. For the coded system the value of E used is rE_b, since the energy for the uncoded bits is spread among the more numerous coded bits.

III. Block Codes

A. BCH Codes

First, we consider BCH codes, which are a special type of block codes, because they can be easily designed for many rates and block lengths. BCH codes are usually denoted by (n, k, t), where n is the block length, k is the number of information bits and t is the number of errors that can be corrected. For block codes, the code rate is equal to k/n.

The approximate bit error rate when a block code is used in an AWGN channel is given by [7]

$$P_b = \frac{1}{n}\sum_{j=t+1}^{n} j \binom{n}{j} p^j (1-p)^{n-j}, \tag{5}$$

where p is the error probability of the channel. For GMSK, p is given by (4).

Using combinations of BCH codes and GMSK modulation schemes that use the same transmission bandwidth, the SNR required to achieve a bit error rate of 10^{-5} is shown in Fig. 2 for various block lengths. As the figure shows, the required SNR can be minimized by choosing the appropriate code and modu-

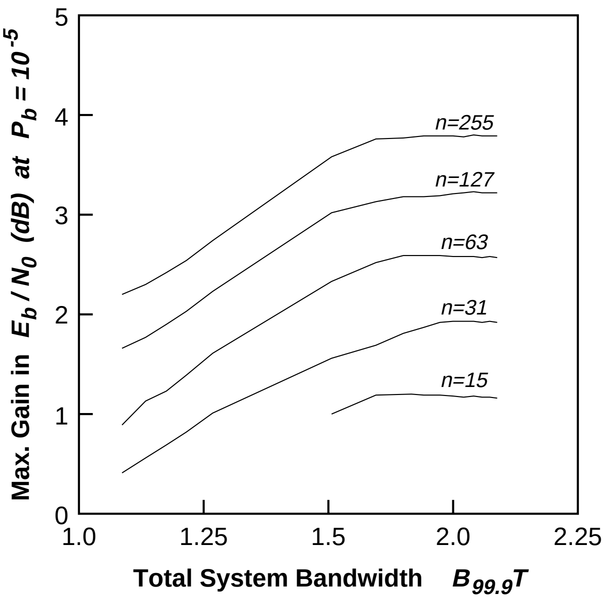

Fig. 3. The maximum gain, relative to the uncoded system, in SNR achieved at a bit error rate of 10^{-5} using BCH codes as a function of the total system bandwidth for various block lengths n.

lation combination. An optimal combination exists because as the code rate is reduced, the gain due to coding increases, but the modulator bandwidth, B_b, that must be used decreases. This results in worse performance for the modulation scheme alone, but the combination of coding and modulation will result in better performance up to a certain point.

The maximum improvement over the uncoded system is approximately $2.5dB$, when a block length of 255 is used. We can also see that the improvement in performance as the block length is doubled is approximately constant. The reason that there are more points as the block length increases is due to the fact that the number of BCH codes that can be designed increases.

If we repeat these calculations for various system bandwidths, a set of curves similar to Fig. 2 results. Looking at the maximum gain achieved over the uncoded system, we obtain the set of curves shown in Fig. 3, where the maximum gain that can be achieved for each block length is shown as a function of the total system bandwidth. As the figure shows, the gain that is obtained increases as the total system bandwidth is increased. The gain starts to saturate around $B_{99.9}T = 2$, but this is only a temporary phenomenon. If the system bandwidth is increased further, the performance can be improved by using a lower rate code, because the performance of the modulation alone is almost constant for $B_bT > 1$.

Now, if we look at the coding rate at which the maximum gain is achieved, i.e., the best performance of the coded GMSK system, we obtain the set of curves shown in Fig. 4. We can see that as the total system bandwidth is increased, the code rate at which the best performance is obtained decreases. There is some variation as the block length is varied, but this can be attributed to the lack of codes for short block lengths.

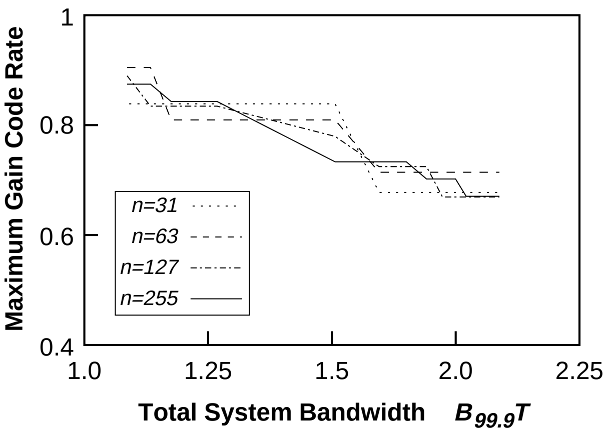

Fig. 4. The code rate at which the maximum gain is achieved for BCH codes as a function of the total system bandwidth for various block lengths n.

Fig. 4 also shows us the optimal balance between coding and modulation as a function of the system bandwidth. For example, when the total system bandwidth is $B_{99.9}T = 2$, the code rate at which the best performance is achieved is approximately $2/3$. This means that using $2/3$ of the bandwidth for modulation and the remaining $1/3$ of the bandwidth for coding is the optimal balance between modulation and coding for GMSK with this system bandwidth. We can also see that more bandwidth should be used for modulation when the system bandwidth is small. On the other hand, when the system bandwidth is large, more of the bandwidth should be used for coding.

B. Perfect Codes

BCH codes are only one example of linear block codes. Here, we find the best performance that can be achieved for coded GMSK within the class of linear block codes. For the coding scheme used in this paper, the best performance is achieved by the code which has the maximum possible rate for a given error correction capability, i.e., the code with maximum k/n for a given t. This kind of code is a perfect code [8].

A perfect code is one that satisfies the Hamming bound with equality, i.e., for binary codes, satisfies

$$n - k = \log_2 \sum_{j=0}^{t} \binom{n}{j} . \qquad (6)$$

This gives us the minimum value of $n - k$ for a given value of t and therefore the maximum value of k/n possible. Given this value of k/n, we can calculate

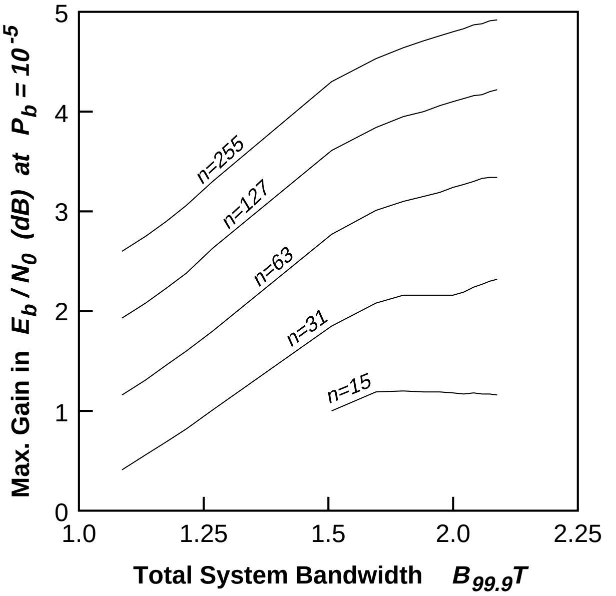

Fig. 5. The maximum gain, relative to the uncoded system, in SNR achieved at a bit error rate of 10^{-5} using ideal block codes as a function of the total system bandwidth for various block lengths n.

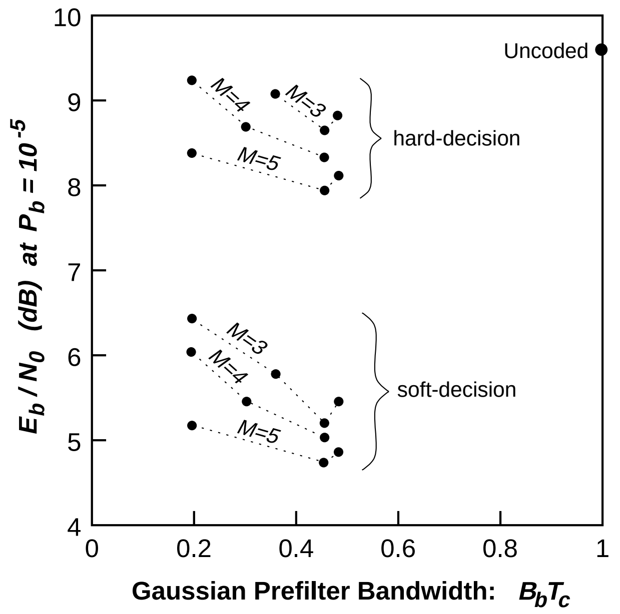

Fig. 6. The SNR required to achieve a bit error rate of 10^{-5} as a function of the normalized Gaussian prefilter bandwidth, B_bT_c, for various convolutional codes with M memory elements decoded using hard and soft decision Viterbi decoding. The total system bandwidth is equal to $B_{99.9}T = 2.05$, which corresponds to an uncoded system with $B_bT = 1.0$.

the coding gain that would be achieved if such a code existed. The results are plotted in Fig. 5.

Comparing these results to the gain for BCH codes in Fig. 3, we can see that the perfect codes provide approximately $1dB$ more gain at higher values of total system bandwidth. Also, the saturation in gain that we saw for BCH codes is not present. Thus, we can see that BCH codes are not the most efficient for low code rates.

IV. Convolutional Codes

A. Code Definitions

Next, we consider coding of GMSK with convolutional codes. Convolutional codes are usually denoted by their code rate, $r = k/n$, constraint length, K, total number of memory elements, M, and free distance, d_{free}. For convolutional codes, k and n are the number of inputs and outputs, respectively, in the encoder.

The convolutional codes that we use are those given in [8, p.285] and [9, p.496], which have the largest d_{free} for a given K and M. These codes provide us with the best performance for convolutionally encoded GMSK systems. We use rates equal to 1/2, 4/7, 3/5, 2/3, 3/4 and 4/5, but codes are not tabulated for all values of K.

In order to use codes that are in some sense equivalent, we group the convolutional codes into sets according to the value of M. This means that all the codes in each set have the same number of states in the decoder trellis and hence the same decoding complexity.

B. Performance

The bit error rate performance of convolutional codes can be approximated by [8]

$$P_b = \frac{1}{k} b_{d_{free}} Z(d_{free}), \qquad (7)$$

where $b_{d_{free}}$ is the total number of information bits associated with code sequences of weight d_{free} and $Z(d_{free})$ is a function of the channel and decoding method. For AWGN channels and hard-decision decoding, i.e., binary symmetric channels with crossover probability p, $Z(d_{free})$ is given by

$$Z(d_{free}) = \left[2\sqrt{p(1-p)}\,\right]^{d_{free}}. \qquad (8)$$

For soft-decision decoding in AWGN channels,

$$Z(d_{free}) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{d_{min}^2 d_{free} E}{2N_0}}\right), \qquad (9)$$

where d_{min}, E and N_0 are as in (4).

As with block codes, we calculate the value of SNR required to achieve a bit error rate of 10^{-5} for both hard and soft-decision Viterbi decoding. The results when the total system bandwidth is 2.05 is shown in Fig. 6. From this figure, we can see that the performance improves in relatively constant intervals as the total number of encoder memory elements, M, is increased. We can also see that hard-decision decoding is roughly $3dB$ worse than soft-decision decoding.

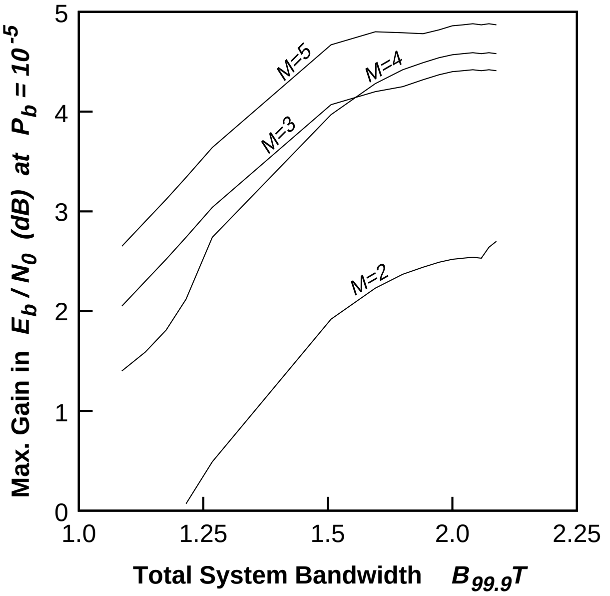

Fig. 7. The maximum gain in SNR achieved at a bit error rate of 10^{-5} for soft-decision decoded convolutional codes as a function of the total system bandwidth for various numbers of memory elements M.

As was the case for block codes, the performance has an optimal value due to the tradeoff in modulation performance and coding gain.

If we focus on soft-decision decoding and compute the performance gain over the uncoded system as a function of the total system bandwidth, we obtain the results shown in Fig. 7. As was the case for block codes, we see that the gain in performance improves as the total system bandwidth is increased.

C. Comparison to Block Codes

Looking at the code rate which achieves the maximum gain in performance, we find that similar results are obtained for convolutional and block codes. As shown in Fig. 8, the optimal code rates decrease as the system bandwidth increases. Due to the small number of codes and corresponding rates, the curve for convolutional codes is lower at smaller system bandwidth values, but the trend and code rates are comparable to the block codes. This shows that the best tradeoff between modulation and coding is approximately independent of the type of code used.

V. Conclusions

In this paper, BCH and convolutional coding for Gaussian Minimum Shift Keying was examined. It was found that the bit error rate performance can be easily improved by more than $3dB$ over an uncoded system with the same bandwidth. Also, the optimal code rate as a function of the total system bandwidth was found. This result showed that the percentage of the system bandwidth that should be used for cod-

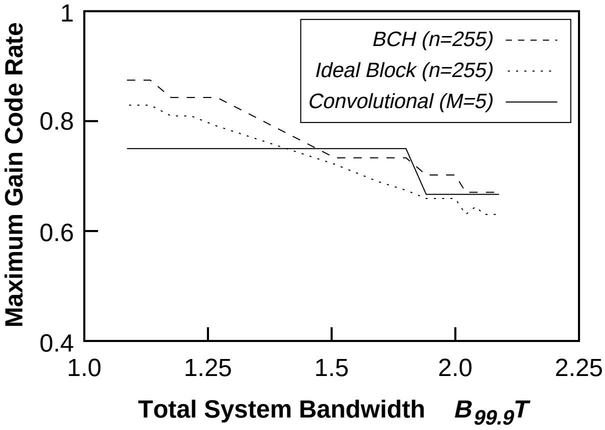

Fig. 8. The code rate at which the maximum gain is achieved as a function of the total system bandwidth for different types of codes.

ing is small when the system bandwidth is small and large when the system bandwidth is large. For a total system bandwidth equal to twice the bit rate, it was found that 1/3 of the system bandwidth should be used for coding and 2/3 for modulation.

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