# Enhanced Conceptual Design Formulae for Frequency Channel Double Reuse Digital Systems using Sectored Cells

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*Abstract*—**To enhance frequency utilization efficiency, cellular mobile radio systems widely use a sector cell structure for the base stations. In this paper, the conceptual design formula previously reported are extended for frequency double reuse indoor picocellular systems. A three sector cell structure is adopted for the urban cellular base stations and omni-directional base stations are used for the indoor system, which is in the final stage of implementation. In particular, the small cluster size case is analyzed for TDMA cellular systems.**

## I. Introduction

The previously proposed frequency double reuse (DR) technique, which allows urban frequency channels to be also used in indoor wireless PBX systems [1][2], makes it more convenient to be able to access both urban and indoor systems with a single handset. Moreover, compared to conventional systems, which use adjacent frequency bands, and dual band systems, which have two sets of wireless circuits built into a single handset, it is possible to design a system with higher spectral efficiency.

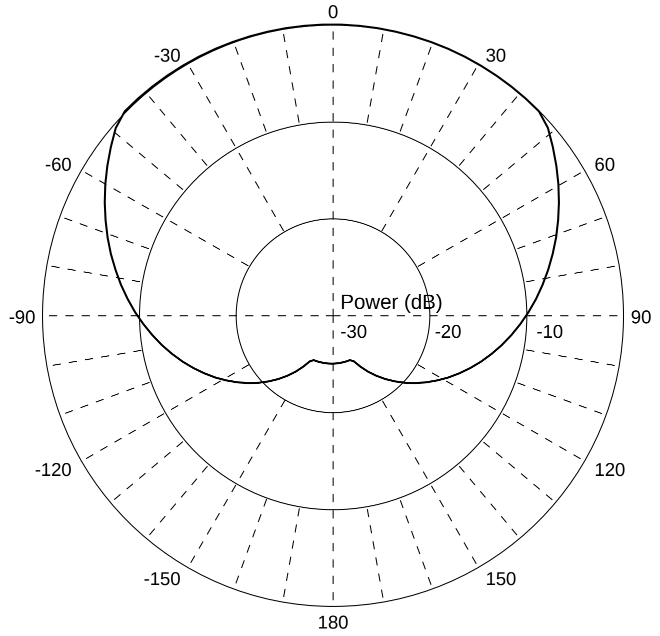
Analog and digital frequency DR systems have already been standardized (IS-94 and IS-136) and are in use in the United States [3]. In Japan, NTT Docomo proposed the standardization of a digital system, compatible with PDC (Personal Digital Cellular), and the standardization process is being carried out by the Association of Radio Industry Businesses (ARIB).

In results reported to date on DR systems, the focus has been on the design of a worst case scenario using a final stage model, which assumes that all possible picocells have been deployed. Using this model, the design formula was derived for the case when the self-interference is the same for both urban and indoor systems.

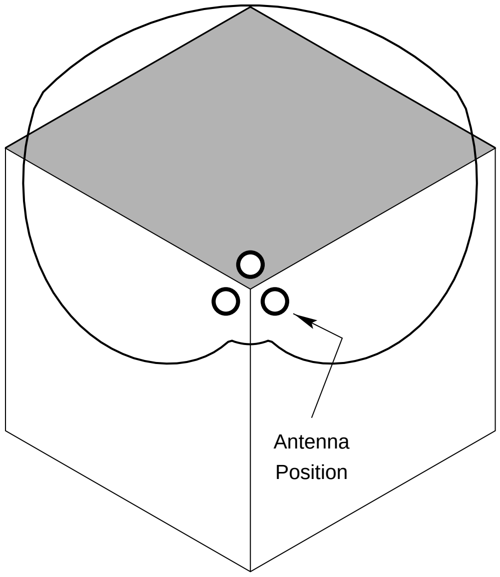
Recent digital cellular systems, such as PDC, GSM, E-TDMA and IS-54 use 3-sector antenna base stations and a cluster size of less than 7. An enhanced conceptual design is needed in these circumstances. In this paper, we extend the previously reported design formula [4] to base stations that use sector cells.

## II. Sector Cell Base Station

The directional antenna's radiation pattern and the sector cell structure of a typical urban mobile telephone system's base station is shown in Fig. 1 [5].



(a) Antenna Radiation Pattern



(b) Sector Cell Structure

Fig. 1. The antenna radiation pattern and sector cell structure of a typical sector cell base station.

The antenna pattern that we use in this paper is given by [6]

$$G_t(\theta) = \begin{cases} 0 & |\theta| \le \theta_1 \\ -0.23 \times (\theta_1 - |\theta|) & \theta_1 \le |\theta| \le \theta_2 \\ -25 & |\theta| \ge \theta_2 \end{cases} \qquad (1)$$

$$\theta_1 = 47^\circ, \theta_2 = 156^\circ.$$

Using this sector cell structure, we extend the conceptual formula for frequency double reuse systems in previous work [4].

The basic design criterion for DR systems is that the signal to interference ratios (SIRs) of the urban and indoor cellular systems that use the same frequency are the same. In mathematical terms, this condition can be written as

$$\frac{S_1}{I_1 + J_{21}} = \frac{S_2}{I_2 + J_{12}}, \qquad (2)$$

where $S_1$ and $S_2$ are the signal powers at the macrocell and picocell edges, respectively, $I_1$ and $I_2$ are the self cochannel interferences for the urban (macrocell) and indoor (picocell) systems, respectively, [1] $J_{21}$ is the mutual cochannel interference that the indoor system causes on the urban system and $J_{12}$ is the mutual cochannel interference that the urban system causes on the indoor system. This criterion means that the service quality is the same for both systems. The SIRs for the picocell and macrocell systems are worst at the edges of the cells, so the design equations are derived when (2) is satisfied at the cell edges.

It is well known that the mean signal power at a distance $r$ from an antenna in a fading environment is given by

$$S_i = K_i \frac{P_i}{r^{\alpha_i}}, \qquad (3)$$

where $P_i$ is the transmitter power, $\alpha_i$ is an attenuation parameter and $K_i$ is a constant given by

$$K_i = \left(\frac{\lambda}{4\pi}\right)^2 G_{ti} G_{ri} \Lambda_i^{\alpha_i - 2}. \qquad (4)$$

In this equation, $\lambda$ is the wavelength of the transmitted wave, $G_{ti}$ and $G_{ri}$ are the antenna gains and $\Lambda_i$ is the line of sight distance from the antenna, i.e., the distance over which $\alpha = 2$. In this paper, as in previous work, we use $\alpha_1 = 3.5$ and $\alpha_2 = 4.5$ [7].

In this paper, we use a directional antenna for the urban system and an omnidirectional antenna for the indoor system. Therefore, the transmitter antenna gain for the urban system, $G_{t1}$, is given by (1), while the the corresponding gain for the indoor system is given by $G_{t2} = 1$.

## III. Final Stage Model

The model for the final stage of implementation of indoor cells is shown in Fig. 2. The interference between indoor and urban systems is greatest in this case and therefore represents a worst case situation. As such, this model is used to derive the conceptual design formula.

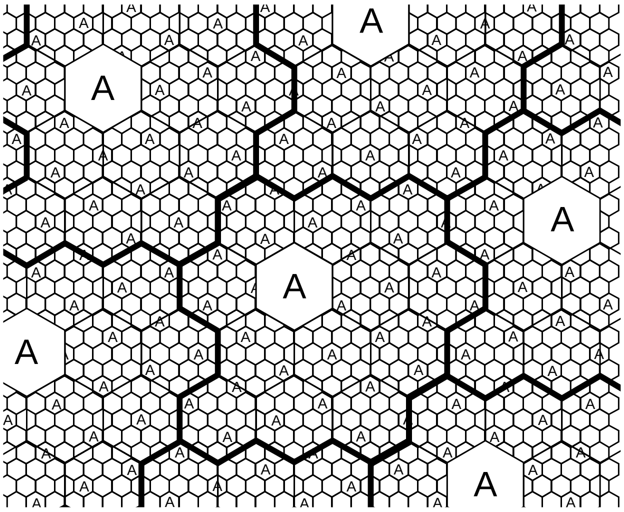


Fig. 2. Final stage model – all possible indoor cells have been deployed.

The "white" urban cells in Fig. 2 do not contain indoor cells that use frequency 'A' because the interference from the urban base station is too great. However, other frequencies can be used. The number of "white", or guard, cells is denoted by $C_g$. In this paper we only consider the case when $C_g = 1$.

In Fig. 2, the urban system has a cluster size of 13, which is equal to the number of frequencies that can be used. In general the cluster size is given by

$$C(i,j) = i^2 + j^2 + ij, \qquad (5)$$

where $i$ and $j$ are shift parameters that show the location of a cell using the same carrier frequency in a hexagonal coordinate system.

We consider the case when the indoor cellular system is introduced after the urban cellular system is completed. In this situation, the urban macrocell radius, $R_1$, and transmitter power, $P_1$, are given then the indoor picocell radius, $R_2$, and transmitter power, $P_2$, are designed.

As in previous work [4], we assume that the communication quality of the urban and indoor systems is the same when they are isolated, i.e.,

$$\frac{S_1}{I_1} = \frac{S_2}{I_2}. \qquad (6)$$

This allows us to rewrite (2) as

$$\frac{J_{21}}{S_1} = \frac{J_{12}}{S_2} \qquad (7)$$

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[1] In this paper, the subscript "1" refers to the urban system, while the subscript "2" refers to the indoor system.

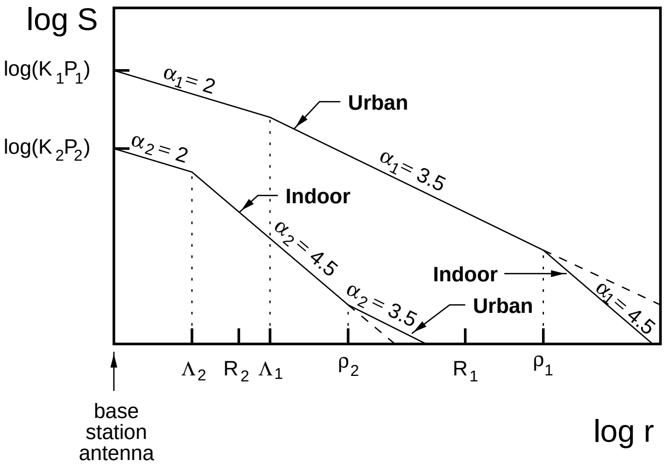

Fig. 3. The signal propagation model used.

and derive the power ratio

$$\frac{P_2}{P_1} = \left(\frac{R_2}{R_1}\right)^{\frac{\alpha_2}{2}+1} \left(\frac{\sqrt{\rho_1 \rho_2}}{\Lambda_2}\right)^{\alpha_2-\alpha_1} \cdot \left(\frac{\Lambda_1}{\Lambda_2}\right)^{\alpha_1-2} E(C_2, C_g). \tag{8}$$

Here, $\rho_1$ and $\rho_2$ are new parameters that tell when the propagation changes from an indoor to an urban environment and vice-versa as shown in Fig. 3. The function $E(C_2, C_g)$ depend only on the cell structure and $\alpha_i$. This function only needs to be calculated once and tabulated, after which the design parameters can be easily calculated.

## IV. EFFECT OF SMALL CLUSTER SIZES

The cluster size, $C_1$, of typical TDMA systems is shown in Table I. All systems listed use cells divided into 3 sectors [8].

TABLE I
CLUSTER SIZES OF CELLULAR SYSTEMS

| Region | System | Cluster Size |
|---|---|---|
| Japan | PDC | 4 |
| Europe | GSM | 3 |
| USA | IS-54 | 7 |

However, for small cluster sizes, the assumption in (6) needs to be reexamined. In previous work, we used theoretical cluster sizes given by

$$C_2 = 0.71418 C_1^{7/9} \tag{9}$$

and shown in Table II. In reality, non-integer cluster sizes as shown in the table do no exist, so in this paper we use cluster sizes, $C_2$, that are integer values close to the theoretical ones.

TABLE II
NON-INTEGER INDOOR CLUSTER SIZES $C_2$

| $C_1$ | Omnidirectional Urban Antenna | Sector Cell Urban Antenna |
|---|---|---|
| 3 | 1.68 | 2.68 |
| 4 | 2.10 | 3.28 |
| 7 | 3.24 | 5.04 |
| 9 | 3.95 | 6.17 |
| 13 | 5.25 | 8.10 |

When integer values for $C_2$ are used, the conceptual design formula becomes

$$\frac{P_2}{P_1} = X_s + \sqrt{X_s^2 + \left(\frac{P_{20}}{P_1}\right)^2}, \tag{10}$$

where $P_{20}/P_1$ is a solution of (8) and $X_s$ is given by

$$X_s = \left(\frac{\bar{I}_1 - \bar{I}_2}{2\bar{J}_{21}}\right) C_2 \left(\frac{R_2}{R_1}\right)^2 \left(\frac{\rho_2}{\Lambda_2}\right) \left(\frac{\Lambda_1}{\Lambda_2}\right)^{\alpha_1-2}. \tag{11}$$

In (11), $\bar{I}_1$, $\bar{I}_2$ and $\bar{J}_{21}$ are equal to $I_1$, $I_2$ and $J_{21}$ normalized by the cell radius $R_i$, respectively, and represent the amount of interference when $K_i P_i = 1$. These values are calculated in the same way as in previous work [4]. The computation results are shown in the appendix and we see that the interference $S_1/I_1$ is about $4dB$ greater for sector cells when the same urban cluster size $C_1$ is used.

We check that when the correction factor $X_s = 0$, (10) is equal to (8). The reason that $X_s$ is not equal to zero is due to the interference imbalance term, $\bar{I}_1 - \bar{I}_2$. The interference imbalance that appears in (11) is most severe in the initial stage of picocell deployment, but its effects will be examined in future work.

## V. SAMPLE CALCULATION

A sample calculation using the enhanced design formula was carried out and the results are shown in Tables III and IV. As in previous work [4], the parameters used for the urban system were $P_1 = 3W$ and $R_1 = 3km$. The propagation parameters used were $\Lambda_1 = 120m$, $\Lambda_2 = 3m$, $\rho_1 = R_1$ and $\rho_2 = R_2$. Also, the value of $R_2$ that resulted from using $P_{20} = 10mW$ for $P_2$ in (8).

Compared to previous results using non-integer cluster sizes [4], the transmitter power of the picocell antenna is larger. This is a result of the interference imbalance that results from the use of realistic integer cluster sizes. This has a great impact on digital cellular systems that use a small cluster size. The use of sector cells reduces this effect to a great extent as can be seen by the much smaller transmitter power $P_2$ for small cluster sizes in Table III.

Table III
Sample Calculation for
Sector Cell Urban Antennae

| $C_1$ | $C_2$ | $R_2(m)$ | $P_2(mW)$ |
|---|---|---|---|
| 3 | 3 | 20.7 | 16.2 |
| 4 | 4 | 20.2 | 17.5 |
| 7 | 7 | 19.9 | 15.3 |
| 9 | 7 | 18.9 | 11.4 |
| 13 | 9 | 18.3 | 10.7 |
| 16 | 12 | 17.6 | 12.6 |
| 19 | 12 | 17.6 | 10.3 |

Table IV
Sample Calculation for
Omnidirectional Urban Antennae

| $C_1$ | $C_2$ | $R_2(m)$ | $P_2(mW)$ |
|---|---|---|---|
| 3 | 3 | 22.2 | 116 |
| 4 | 3 | 21.9 | 48.9 |
| 7 | 4 | 20.9 | 18.6 |
| 9 | 4 | 20.4 | 10.3 |
| 13 | 7 | 19.7 | 14.8 |
| 16 | 7 | 19.2 | 11.4 |
| 19 | 9 | 18.9 | 11.4 |

## VI. Conclusions

In this paper, the conceptual design formula for frequency double reuse systems was enhanced so that it could be applied to urban TDMA systems with small cluster sizes. The formula was also modified to consider realistic integer cluster sizes and it was found that an interference imbalance occurs. The use of sector cells was found to reduce the effects of this imbalance considerably.

In future work, the effects of this imbalance for the initial deployment stage of indoor cells will be examined.

## Appendix
## Computation Results

The parameters defined in previous work [4] were calculated for omnidirectional and sector cell antennae and the results are tabulated in tables V and VI respectively. In both cases, $C_g = 1$, $\alpha_1 = 3.5$ and $\alpha_2 = 4.5$. Note that these parameters only depend on the cell structure and $\alpha_i$.

Table V
Computation Results for Omnidirectional Antennae

| $C_1$ | $C_2$ | $\bar{I}_1$ | $\bar{I}_2$ | $\bar{J}_{21}$ | $E$ | $S_1/I_1$ (dB) |
|---|---|---|---|---|---|---|
| 3 | 3 | 0.188 | 0.0508 | 1.095 | 1.290 | 7.26 |
| 4 | 3 | 0.113 | 0.0508 | 1.170 | 1.365 | 9.47 |
| 7 | 4 | 0.0426 | 0.0266 | 1.240 | 1.625 | 13.7 |
| 9 | 4 | 0.0274 | 0.0266 | 1.256 | 1.777 | 15.6 |
| 13 | 7 | 0.0144 | 0.00754 | 1.269 | 2.036 | 18.4 |
| 16 | 7 | 0.0100 | 0.00754 | 1.273 | 2.203 | 21.3 |
| 19 | 9 | 0.0074 | 0.00429 | 1.276 | 2.352 | 21.1 |

Table VI
Computation Results for Sector Cell Antennae

| $C_1$ | $C_2$ | $\bar{I}_1$ | $\bar{I}_2$ | $\bar{J}_{21}$ | $E$ | $S_1/I_1$ (dB) |
|---|---|---|---|---|---|---|
| 3 | 3 | 0.06535 | 0.05076 | 1.0953 | 1.6647 | 11.9 |
| 4 | 4 | 0.04150 | 0.02657 | 1.1695 | 1.8545 | 13.8 |
| 7 | 7 | 0.01580 | 0.00754 | 1.2404 | 2.3778 | 18.0 |
| 9 | 7 | 0.01004 | 0.00754 | 1.2555 | 2.3624 | 20.0 |
| 13 | 9 | 0.00544 | 0.00429 | 1.2685 | 2.6643 | 22.6 |
| 16 | 12 | 0.00367 | 0.00224 | 1.2729 | 3.0708 | 24.4 |
| 19 | 12 | 0.00265 | 0.00224 | 1.2755 | 3.0676 | 25.8 |

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