The acoustics of Japanese wooden drums called "mokugyo"[a]

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A drumlike traditional Japanese instrument, the mokugyo, is experimentally discussed. First, the acoustic characteristics of 176 mokugyos with diameters ranging from 7.5 to 120 cm and three drumsticks were measured. Results show that (a) the sound spectra consist of two common peaks [F_1 (Hz): first peak frequency, F_2 (Hz): second peak frequency] close together, with an average ratio (F_2/F_1) of 1.15, and (b) a drumstick beating the mokugyo is translated into an impact force applied over a period of time from 1 to 6 ms related to the mass and stiffness of the material wrapped around the tip of the drumstick. Second, to evaluate the acoustic response of a mokugyo in the final tuning process, the mechanical and acoustical analogy between the mokugyo and a bass reflex loudspeaker is theoretically and experimentally discussed. Results show that the model can be estimated within a relative error of 0.52% from the mass of wood chips. Finally, from a psychological experiment, the timbre of the mokugyo shows higher scores on psychological scales when the ratio (F_2/F_1) becomes 1.15.
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I. INTRODUCTION

In most of the rituals for Buddhism, the sutras are recited to the rhythm created by several percussion instruments such as a mokugyo, a keisu, and a large bowl-shaped gong.[1] A mokugyo is a round drum made of wood used during Zen rituals like the recitation of the sutras. Buddhists recite sutras while striking a mokugyo with a regular rhythm. Most Japanese have been familiar with the mokugyo sound since childhood. The word, "mokugyo," means "a wooden fish" in Japanese. In Buddhism the fish, which never sleeps, symbolizes wakefulness. It was introduced from China to Japan around the 17th century. From an acoustic engineering standpoint, the mokugyo has never been investigated.

In recent years, physical modeling of the sound production mechanism of percussion instruments like a mokugyo has been studied. For example, Chaigne *et al.*[2,3] proposed a physical model of xylophone sound production and synthesized xylophone sounds based on the proposed model. They reported that the synthesized sound was close to the real sound of the xylophone, and that the quality of the synthesized sound could be controlled by adjusting the physical structure of the xylophone using the proposed model. L. Rhaouti *et al.*[4] also proposed a physical model of the kettledrum and its synthesized sounds from a similar standpoint. On the other hand, Obata *et al.*[5] investigated acoustic characteristics of a Japanese traditional percussion instrument, the "ôdaiko." They reported that the fundamental frequency component of ôdaiko sounds involved approximately 25 beats per second. The beats were produced by a coupled vibration between the drumskins on both sides of the body and the enclosed air. In the same way, Ando *et al.*[6,7] investigated and synthesized the sound of the "tudumi," and Aoki *et al.*[8] investigated that of the "washo."

Figure 1(a) shows the front view of a general mokugyo with a 37.5-cm diameter, (b) shows its side view with a 32.0-cm height, and (c) shows a drumstick with a 43.0-cm length. The body is made of camphorwood with a density of 0.16 g/cm^3. Reliefs of dragons or fishes are carved on the body. The sound is produced by striking the contact area with a drumstick. As indicated by the dotted line in Fig. 1(b), the body is carved from a single block of wood to have a large internal cavity, which is quite a feat, given its narrow slit opening (intended to resemble the mouth of a fish). The cavity walls are 2.5 cm thick on average. These components form a Helmholtz resonator system. The handle of the drumstick is made of wood, and the tip is wrapped in rubber.

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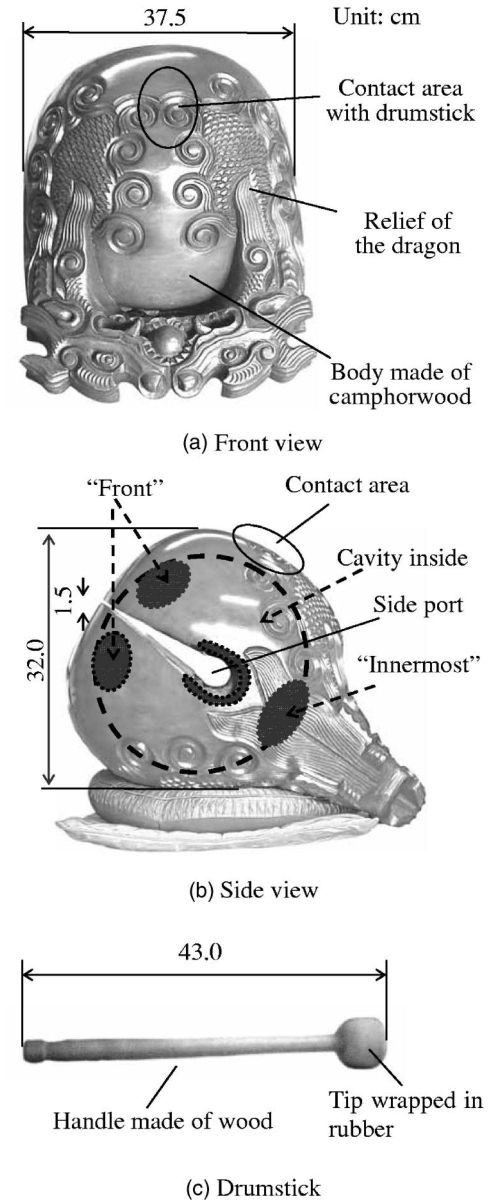

FIG. 1. The mokugyo. (a) Front view. (b) Side view. (c) The drumstick.

Typical drumstick tips are often wrapped in thread, cloth, or leather. The kinds of wrapping materials may be an important factor for producing beautiful timbre, since the timbre varies with the material. Therefore, the artisan making a mokugyo also makes a drumstick exclusively for the mokugyo.

Traditional skills for making mokugyo are endangered, since the number of artisans has decreased. One of the causes is the technical difficulty involved. The most difficult process is the adjustment of the timbre. This is done by chiseling the cavity of the side port little by little based only on the artisan's intuition. This process suggests that the physical characteristics which determine the timbre are affected by various structural elements of the body. If it is possible to control such physical parameters numerically using a physical model, the desired timbre could be produced easily.

In this paper, acoustic characteristics of mokugyos are investigated experimentally, theoretically, and psychoacoustically. In Sec. II, five physical characteristics of the mokugyo and its drumstick are measured and discussed: mokugyo sound spectrum, vibration of the body, waveform and spectrum of the driving forces of the drumstick, directivity, and sound intensity vector of the acoustic radiation of the mokugyo. In Sec. III, a physical model of the mokugyo is proposed in which it is assumed that the mechanical vibratory section of the body and the Helmholtz resonator system are coupled. The mokugyo sound is simulated in the frequency domain. Parameter values used in the proposed model are estimated by comparing the spectrum of the simulated sound with that of the original sound. In Sec. IV, the proposed model is evaluated. Three areas in the cavity, which are called "front," "innermost," and "side port," are chiseled, and then the difference in the spectrum before and after the chiseling is measured and evaluated. In Sec. V, the relation between favorable timbre and physical characteristics of the mokugyo is experimentally examined with the semantic differential method in order to establish indices for evaluating mokugyo timbre.

II. EXPERIMENTS

In order to investigate the physical and acoustic characteristics of the mokugyo, five characteristics were measured: sound spectrum, vibration of the body, drumstick driving force, directivity, and sound intensity vector.

A. Sound spectrum

1. Experimental method

One hundred seventy-six mokugyos were used for this measurement. The minimum diameter of the mokugyos was 7.5 cm and the maximum was 120 cm with an average diameter of 46 cm. Each mokugyo was struck with a drumstick at the contact area shown in Fig. 1. This experiment was conducted in an anechoic room with a volume of 60 m^3. To record the sounds a microphone was set at a distance of 100 cm away from the side port, and the output signals were recorded on DAT (Technics: SV-MD11) with a sampling frequency of 48 000 Hz. The recorded sounds were analyzed using a (fast Fourier transform) (FFT) with a rectangular window of 32 768 samples.

2. Results

Figure 2 shows waveforms and spectra for three mokugyos with typical characteristics. Time scales in the figures are different from each other. The rising edge of the waveform (i.e., when the drumstick contacts the body) is set to "0" in the time scale. The beats were observed for all three waveforms whose periods are (a) 7.2 ms for the mokugyo with a diameter of 9 cm, (b) 46 ms for that with a diameter of 37 cm, and (c) 110 ms for that with a diameter of 90 cm. Three peaks were observed in all sound spectra. The two peaks at lower frequencies are significant. In this paper, the three peaks are defined as the first, second, and third resonance frequencies, F_1, F_2, and F_3 (Hz) in increasing order. Each resonance frequency decreases as the diameter increases, and F_1 and F_2 are close. These two close peaks are a general physical characteristic for the sound of the mokugyo, since such peaks were observed for 174 out of 176

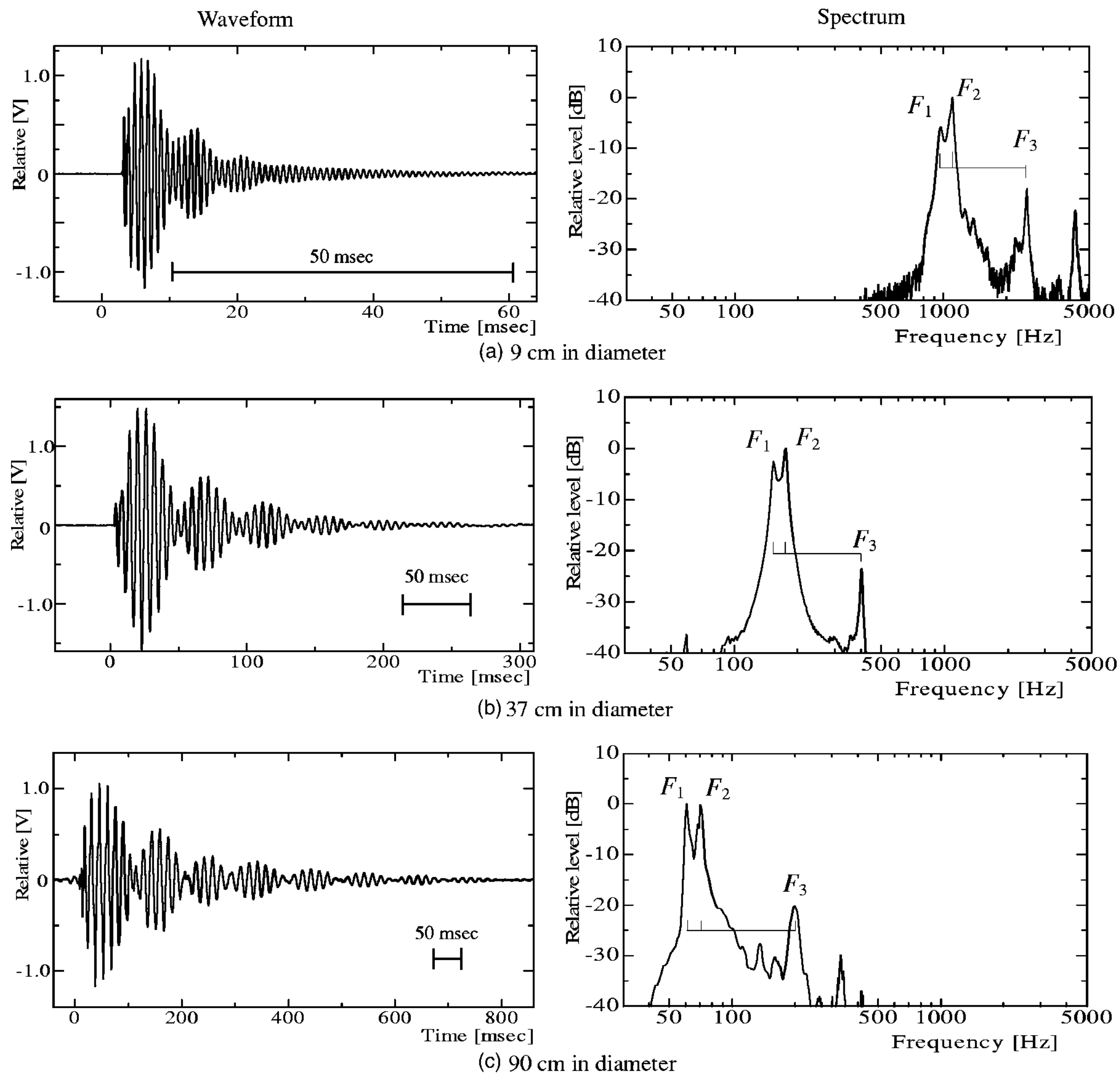

FIG. 2. Waveforms and spectra of three kinds of mokugyo: (a) 9 cm in diameter, (b) 37 cm, and (c) 90 cm.

mokugyos. Figure 3 shows a correlation diagram between F_1 and F_2 for these 174 mokugyos. F_1 and F_2 show a linear correlation, and its correlation coefficient is 0.997. The average ratio of F_1 and F_2 (i.e., the average of F_2/F_1) is approximately 1.15. These results suggest that the period of the beat observed in the waveform is $1000/(F_2-F_1)$ (ms).

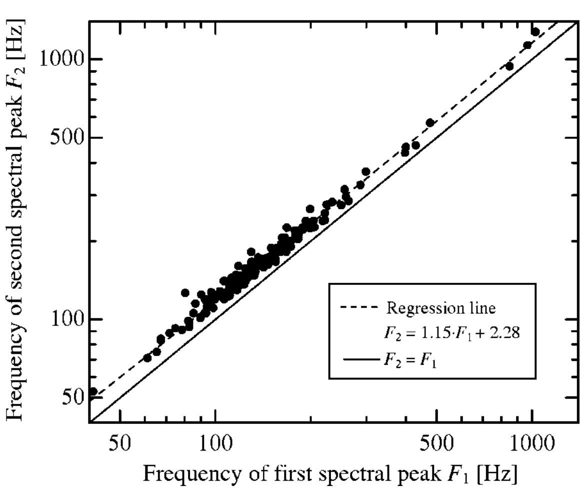

FIG. 3. Correlation between second peak F_2 and first peak F_1 of the sound spectra of 174 mokugyos.

B. Vibration of the body

1. Experimental method

The mokugyo shown in Fig. 2(b) was used in this measurement. The vibration of the driving point of the body was measured using an acceleration pickup (RION: PV-90V) set 2 cm above the contact point for the drumstick along with the surface. The mokugyo was struck with a drumstick at the contact area shown in Fig. 1. The output acceleration signal was converted to a velocity signal via a charge amplifier (SHOWA: 4006).

2. Results

Figure 4 shows the waveform of the vibration velocity of the body, in which the rising edge of the waveform (i.e., when the drumstick contacts the body) is set to "0" in the time scale. A beat was observed in the waveform. The period

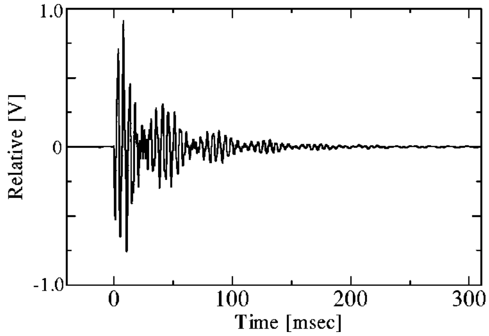

FIG. 4. Vibration velocity of the driving point of the body.

of the beat, 46 ms, is approximately equal to that of the sound wave in Fig. 2(b). As is seen in Fig. 2(b), the phase difference of the beat between the vibration and the sound is 26 ms, which is the sum of half the period of the beat and the propagation time to the observation point. Half the period was 23 ms, and the propagation time was 3 ms since the observation point for the sound was set at a distance of 100 cm away from the mokugyo. These results suggest that the energy of the vibration is exchanged alternately from the body to the Helmholtz resonator system. This energy exchange is caused by the coupled vibration of the two systems with the same eigenfrequency.

C. Effect of impact forces

1. Experimental method

Three kinds of drumsticks were used. Their tips were wrapped in thread, leather, and rubber. The length and mass of the drumstick with the thread tip were 23 cm and 26 g, those of the drumstick with the leather tip were 40 cm and 173 g, and those of the drumstick with the rubber tip were 55 cm and 250 g. The impact force was measured using an impedance head (RION: PF-60) set on a rigid wall. The impedance head was struck with the drumstick by an experimenter. The experimenter kept the driving force at 170 N through all measurements. The output signal was recorded via a charge amplifier (SHOWA: 4006).

Following the measurements of the impact forces, the sound spectra of a mokugyo with a 42-cm diameter were measured using two kinds of drumsticks: a leather-wrapped drumstick and a thread-wrapped drumstick. The method of recording and analyzing the measured sounds was the same as described in Sec. II A.

2. Results

Figure 5(a) shows the impact forces as a function of time. The force duration varies from 1 to 6 ms according to the material wrapped around the tip. Figure 5(b) shows the spectra of the impact forces. The frequency range varies according to the wrapping material.

Figure 6 shows the sound spectra of a mokugyo struck with two kinds of the drumsticks. In the frequency region less than $F_3 = 517$ Hz, the sound spectrum for the drumstick wrapped in thread corresponds to that for the one wrapped in leather. At higher frequencies the spectrum level for the drumstick wrapped in leather decreases. The result is consistent with the frequency range of the impact force.

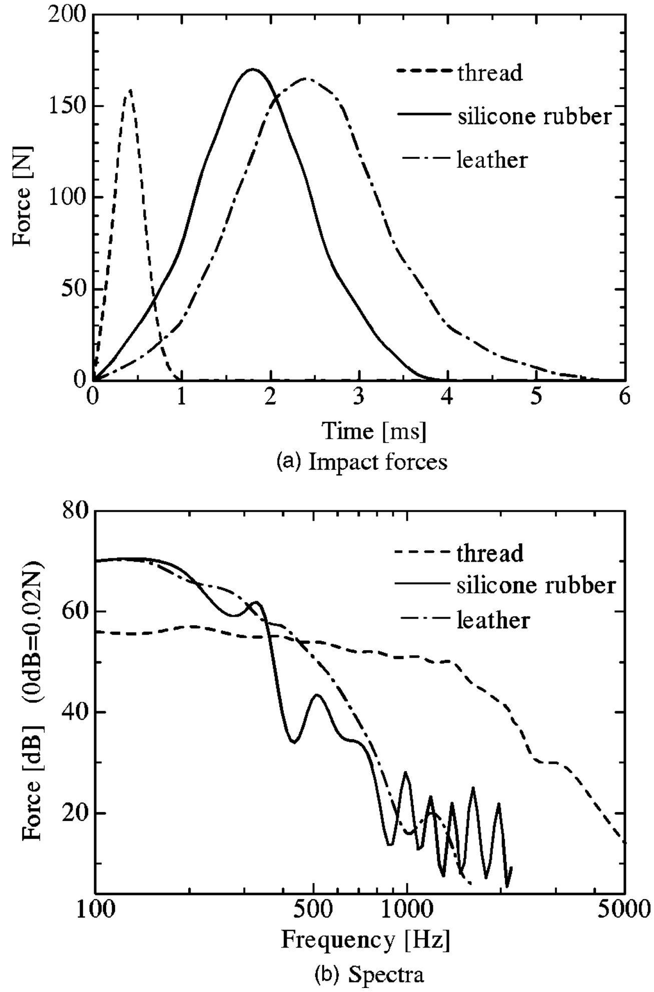

FIG. 5. Waveforms and spectra of impulsive forces from three kinds of drumsticks. (a) Impact forces. (b) Spectra.

D. Directional response

1. Experimental method

The mokugyo shown in Fig. 2(b) was used in this measurement, and its resonance frequencies are $F_1 = 155$ Hz,

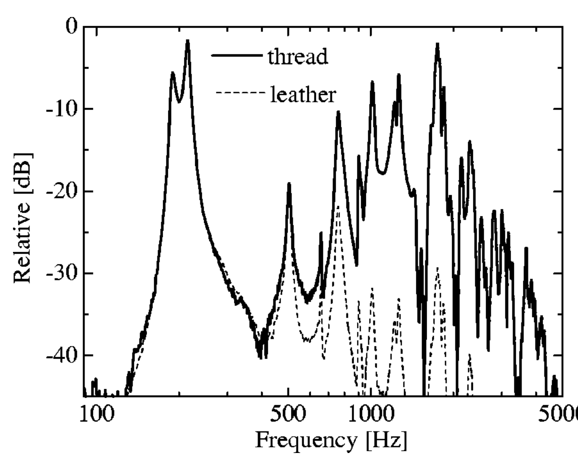

FIG. 6. Sound spectra from two kinds of drumsticks.

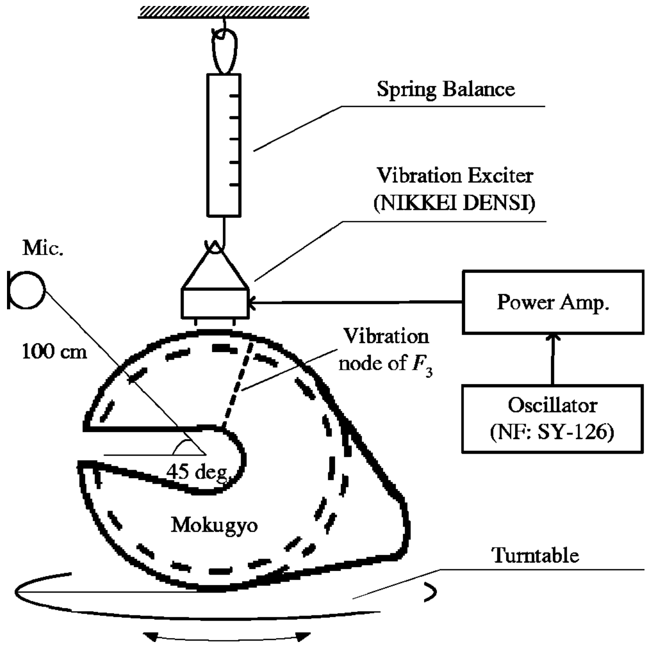

FIG. 7. Block diagram of directivity measurement setup.

$F_2 = 177$ Hz, and $F_3 = 402$ Hz. Figure 7 shows the block diagram of the measurement system. The mokugyo was put on a turntable and driven steadily with sinusoidal waves corresponding to each resonance frequency. The sound pressure level was measured as the turntable rotated. The output signal from the oscillator (NF: SY-126) was amplified and transmitted to a vibration exciter (Nikkei Denshi: Golden Tone) whose mass was compensated for by a spring balance. The body of the mokugyo was excited by the vibration exciter fixed on the contact area. A microphone was set at a distance of 100 cm away from the side port at a 45° angle from the horizontal plane. Also, an acceleration pickup (RION PV-90V) was used to investigate the vibration at various points on the surface of the body. This acceleration signal was recorded via a charge amplifier (SHOWA: 4006).

2. Results

Figure 8 shows the directivity for each resonance frequency. F_1 and F_2 are omnidirectional while F_3 shows a figure-8 pattern. This figure-8 pattern of F_3 was caused by the divided vibration of the body since the vibration node, which divides the upper part of the body in two, was observed as shown in Fig. 7.

E. Sound intensity measurement

1. Experimental method

The mokugyo shown in Fig. 2(b) was used. A two-dimensional sound intensity vector was measured by exciting the body using the same method described in Sec. II D. The measurement plane was a vertical one dividing the body in half. One hundred eighty-five measuring points, which were allocated on the measuring plane at 2-cm intervals around the side port, were used. The sound intensity vector was measured using the two rotating microphone method.[9,10]

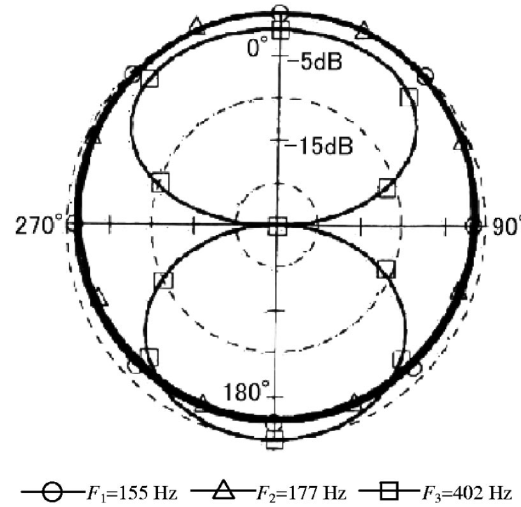

FIG. 8. The directional patterns of three resonant frequencies.

2. Results

Figure 9 shows the sound intensity vector around the side port at frequencies F_1, F_2, and F_3. The arrows and their lengths indicate the directions and the levels (dB) of the vectors. The levels shown in Fig. 9(c) are compensated by +30 dB, since these levels are lower than the others.

For $F_1 = 155$ Hz and $F_2 = 177$ Hz, many of the vectors spread radially from the side port, and no vectors spread directly from the body. Therefore, the vibration mode at F_1 and F_2 would be a simple point sound source corresponding to the vibration of the air in the side port. This result is consistent with the directivity shown in Fig. 8. On the other hand, for $F_3 = 402$ Hz, the vectors from the side port are hardly observed, and most of the vectors are radiated from the body. This result suggests that the figure-8 pattern of F_3 shown in Fig. 8 is due to the divided vibration of the body.

III. THEORETICAL ANALYSIS

As shown in Sec. II, the sound of the mokugyo involves two close resonance frequencies, F_1 and F_2. In this section, the relationship between these two resonance frequencies and the structure of the mokugyo is discussed based on a physical model. This structure composed of the body, cavity, and side port is analogous with that of the bass reflex loudspeaker,[11,12] and a guitar,[13–15] since there is coupling between vibrations of plates and the internal air cavity they enclose. An equivalent model of the mokugyo is proposed based on the theory of the bass reflex loudspeaker. Then, the mokugyo with a diameter of 37 cm shown in Fig. 2(b) is simulated acoustically.

A. Model of the mokugyo

Figure 10 shows the physical model of the mokugyo proposed in this paper, and Table I shows the parameters used in this model. The model is composed of two vibration systems: a mechanical vibration system driven by the drumstick and an acoustic vibration system composed of the cavity and the side port. In the mechanical vibration system, the effective vibration part of the body is assumed to be a circular vibratory board with equivalent mass m_m, which is fixed

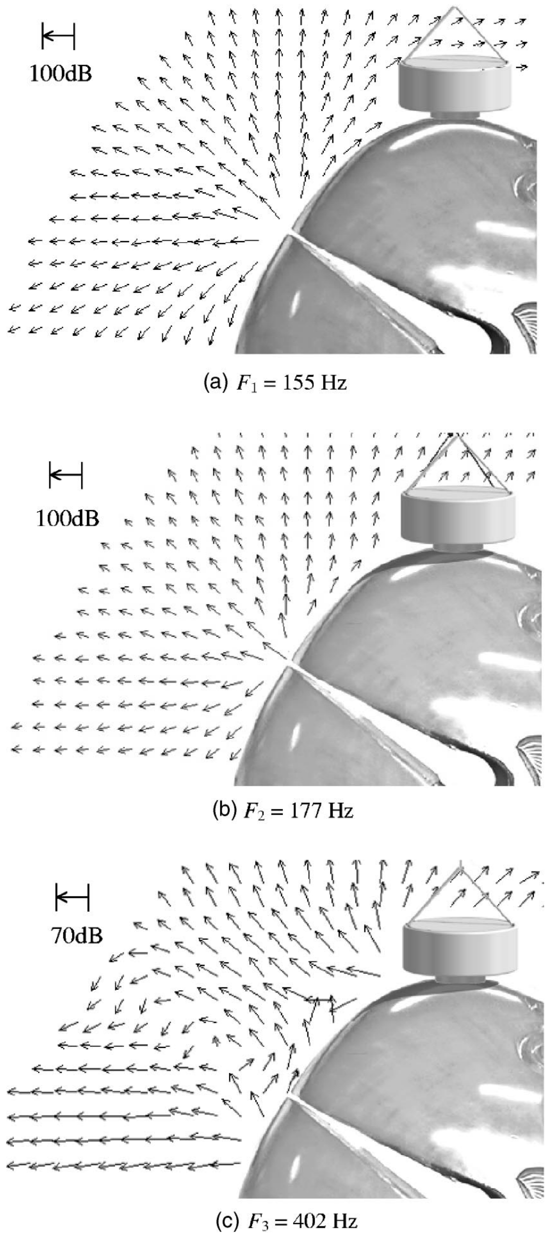

FIG. 9. Sound intensity vectors around the port: (a) $F_1 = 155$ Hz, (b) $F_2 = 177$ Hz, and (c) $F_3 = 402$ Hz.

on a rigid wall. m_m is derived from the sum of the mass of the board, m_d, and an additional mass radiated from the board, m_{ad}. The additional mass m_{ad} is a reaction force from the medium when the board is vibrating, and acts to increase the mass of the board. In this model, furthermore, the deflection forced by the drumstick is modeled using both a spring with equivalent stiffness, s_m, supporting the circular vibratory board and a mechanical resistance, r_m. The board is driven by an external force F due to the drumstick and is vibrated at a velocity v_d. In an acoustic vibration system, the cavity corresponds to the reciprocal of acoustic compliance, i.e., stiffness, s_c. The side port is modeled as a cylindrical hole hollowed out of a rigid wall. The hole is equivalent to the sum of the air mass, m_p, and an additional mass radiated from the side port, m_{ap}. The acoustic resistance of the side port is ignored in this model, since the resistance is much smaller than the mechanical resistance r_m.

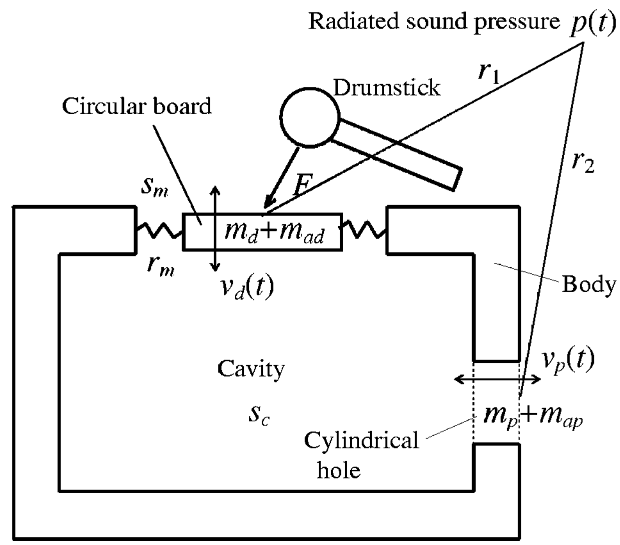

FIG. 10. Physical model of the mokugyo.

Figure 11 shows the electrical circuit equivalent to the model in Fig. 10. Coupling between the mechanical and acoustical systems is replaced by a transformer with a turn ratio equivalent to the area ratio of the effective vibration area of the body, S_d, and the cross-sectional area of the side port, S_p. Converting the secondary side (acoustical system) of the transformer into the primary side (mechanical system), the following simultaneous differential equations can be derived:

TABLE I. Parameter values used in the proposed model.

Variable	Symbol	Value
	Mechanical system	
Effective vibration area of the board	S_d	0.07 m^2
Radius of the board	a_d	14.9 cm
Mass of the board	m_d	5.48 kg
Additional mass of the board	m_{ad}	0.616 kg
Equivalent mass	m_m	$m_d + m_{ad} = 6.10$ kg
Equivalent stiffness	s_m	6.76×10^6 N/m
Equivalent resistance	r_m	210 N·s/m
Vibration velocity of the board	v_d	⋯
	Acoustic system	
Volume of the port	W_p	1.55×10^{-3} m^3
Cross section area of the port	S_p	1.50×10^{-2} m^2
Radius of the port	a_p	6.9 cm
Length of the port	l_p	10.3 cm
Mass of the port's air	m_p	1.87×10^{-3} kg
Additional mass of the port	m_{ap}	2.36×10^{-3} kg
Equivalent mass converted into the mechanical system	m_l	$(S_d/S_p)^2(m_p + m_{ap}) = 0.092$ kg
Volume of the cavity	W_c	1.80×10^{-2} m^3
Equivalent stiffness of the cavity	s_c	1.0×10^5 N/m
Particle velocity	v_p	⋯
	Other parameters	
Impact force with the drumstick	F	170 N
Distance from the board to the observation point	r_1	130 cm
Distance from the side port to the observation point	r_2	100 cm
Air density	ρ_0	1.21 kg/m^3
Sound velocity	c	343 m/s

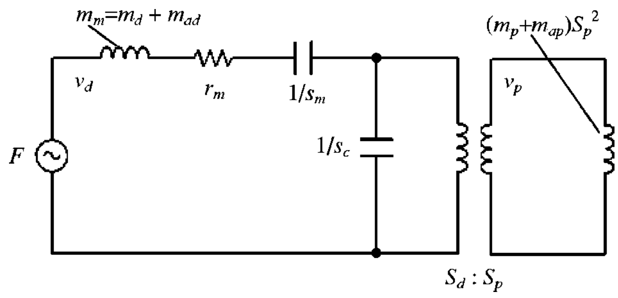

FIG. 11. Equivalent circuit of the physical model.

$$m_m \frac{d}{dt} v_d(t) + r_m v_d(t) + s_m \int v_d(t) dt + s_c \int \left\{ v_d(t) - \frac{S_p}{S_d} v_p(t) \right\} dt = F(t), \tag{1}$$

$$m_l \frac{S_p}{S_d} \frac{d}{dt} v_p(t) + s_c \int \left\{ \frac{S_p}{S_d} v_p(t) - v_d(t) dt \right\} = 0,$$

where $v_d(t)$ denotes the vibration velocity of the circular board, $v_p(t)$ denotes the particle velocity of the side port, and m_l denotes the equivalent mass of the side port converted to the mechanical system. m_l is given by

$$m_l = \left(\frac{S_d}{S_p} \right)^2 (m_p + m_{ap}). \tag{2}$$

The initial conditions of Eq. (1) are

$$v_d(0) = v_p(0) = \int_{-\infty}^{0} v_d(0) dt = \int_{-\infty}^{0} v_d(0) dt = 0. \tag{3}$$

From Eq. (1), the Fourier transforms of $v_d(t)$ and $v_p(t)$, that is, $\hat{v}_d(\omega)$ and $\hat{v}_p(\omega)$, are

$$\hat{v}_d(\omega) = \frac{j\omega(s_c - m_l\omega^2)\hat{F}(\omega)}{m_m m_l \omega^4 - (s_c m_m + m_l s_c + m_l s_m)\omega^2 + s_c s_m - j(m_l r_m \omega^3 - s_c r_m \omega)} \tag{4}$$

and

$$\hat{v}_p(\omega) = \frac{j\omega s_c S_d \hat{F}(\omega)/S_p}{m_m m_l \omega^4 - (s_c m_m + m_l s_c + m_l s_m)\omega^2 + s_c s_m - j(m_l r_m \omega^3 - s_c r_m \omega)}, \tag{5}$$

where ω denotes the angular frequency, which is related to the vibration frequency f by $\omega = 2\pi f$. From the results shown in Sec. II E, the mechanical vibration system in the body and the acoustic center of the side port can be assumed to be simple point sound sources. Thus, the complex sound pressure $\hat{p}(\omega)$ at an arbitrary observation point is given by

$$\hat{p}(\omega) = \frac{j\rho_0\omega}{2\pi} \left\{ \frac{\epsilon^{-jkr_1}}{r_1} S_d \hat{v}_d(\omega) - \frac{\epsilon^{-jkr_2}}{r_2} S_p \hat{v}_p(\omega) \right\}, \tag{6}$$

where r_1 denotes the distance from the sound source of the mechanical vibration system, r_2 denotes that from the side port, ρ_0 denotes the density of air, and k denotes the wave constant given by $k = \omega c$. By applying Eqs. (4) and (5) to Eq. (6), the sound pressure at the observation point is given by

$$\hat{p}(\omega) = \frac{-\rho_0 S_d \omega^2 \hat{F}(\omega)}{2\pi} \left\{ \frac{(s_c - m_l\omega^2)\epsilon^{-jkr_1}/r_1 - s_c \epsilon^{-jkr_2}/r_2}{m_m m_l \omega^4 - (s_c m_m + m_l s_c + m_l s_m)\omega^2 + s_c s_m - j(m_l r_m \omega^3 - s_c r_m \omega)} \right\}. \tag{7}$$

B. Measurement of the model parameters

In this section, parameter values for the proposed model in Sec. III A are calculated using a mokugyo with a 37-cm diameter.

1. Model parameters

Table II shows the profile of the mokugyo used. Assuming the shape of the mokugyo is spherical, the surface area of the hemisphere (0.215 m^2) can be calculated from the diameter of the mokugyo. The area of the circular vibratory board, S_d, corresponding to the effective vibration area of the body in the model is supposed to be $\frac{1}{3}$ the surface area of the hemisphere. When $S_d = 0.07\,m^2$, the radius of the vibratory board, a_d, is 15.1 cm. In the proposed model, since the side port of the body is assumed to be a circular cylinder, its radius, a_p, is 6.9 cm and length, l_p, is 10.3 cm, where the cross section, S_p, of the side port (i.e., the cross section of the circular cylinder) is assumed to correspond to the actual area of the side port opening. The maximum force driven with the drumstick, F_{max}, is set at 170 N as shown in Fig. 5.

TABLE II. Profile of the mokugyo used in the simulation.

Parameter	Actual measurement
Diameter of the mokugyo	37 cm
Aperture area of the side port	1.50×10^{-2} m^2
Capacity of the side port	1.55×10^{-3} m^3
Capacity of the cavity	1.80×10^{-2} m^3

The distance from each source to observation points r_1 and r_2, the density of air, ρ_0, and the sound velocity, c, are set at $r_1 = 130$ cm, $r_2 = 100$ cm, $\rho_0 = 1.21$ kg/m^3, and $c = 343$ m/s.

2. Mechanical system parameters

a. Equivalent mass of the circular vibratory board: m_d. From the original first resonance frequency of the body, F_1, and the resonance frequency, F'_1, produced by fixing an arbitrary additional mass, m'_d, on the surface of the mokugyo, the equivalent mass of the vibratory board, m_d, is given by[11]

$$m_d = m'_d \frac{F_1'^2}{F_1^2 - F_1'^2}. \tag{8}$$

$F_1 = 155$ Hz was used for the mokugyo as shown in Fig. 2(b). Here, F'_1 was measured and found to be 152.2 Hz, when the additional mass ($m'_d = 0.20$ kg) was experimentally fixed on the driving point of the body. Thus, $m_d = 5.48$ kg was obtained from Eq. (8).

b. Additional mass of the board: m_{ad}. The additional mass, m_{ad}, due to the radiation from the board, is derived from the following equation,[12]

$$m_{ad} = \tfrac{16}{3} \rho_0 a_d^3, \tag{9}$$

which expresses an inductive component of the radiated impedance produced by the circular vibratory board.[12] From Eq. (9), $m_{ad} = 0.3269$ kg.

c. Equivalent stiffness: s_m. From F_1 and m_d, the equivalent stiffness of the board s_m (Ref. 11) is

$$s_m = 4\pi^2 F_1^2 m_d. \tag{10}$$

From Eq. (10), $s_m = 6.76 \times 10^6$ N/m.

d. Equivalent resistance: r_m. The quality factor Q_1 of the first resonance frequency is 25.4, since Q_1 is $F_1/\Delta F_Q$,[11] where ΔF_Q is the bandwidth when the sound pressure level becomes $1/\sqrt{2}$ times the sound pressure level at F_1. Thus, the equivalent resistance of the mechanical system r_m (Ref. 11) is

$$r_m = \frac{2\pi F_1 m_d}{Q_1}. \tag{11}$$

From Eq. (11), $r_m = 210$ N·s/m.

3. Acoustical system parameters

a. Stiffness of the cavity: s_c. From the volume of the cavity, W_c, the area of the vibratory board, S_d, the sound velocity, c, and the density of air, ρ_0, the stiffness of the cavity, s_c, is

$$s_c = \frac{\rho_0 c^2 S_d^2}{W_c}. \tag{12}$$

From Eq. (12), $s_c = 3.86 \times 10^4$ N/m.

b. Air mass in the side port: m_p. Multiplying the volume of the side port, W_p, by the density of air, ρ_0, gives $m_p = 1.87 \times 10^{-3}$ kg.

c. Additional mass of the side port: m_{ap}. From Eq. (9), $m_{ap} = 2.13 \times 10^{-3}$ kg. This value was obtained in the same way as the additional mass of the vibratory board, m_{ad}.

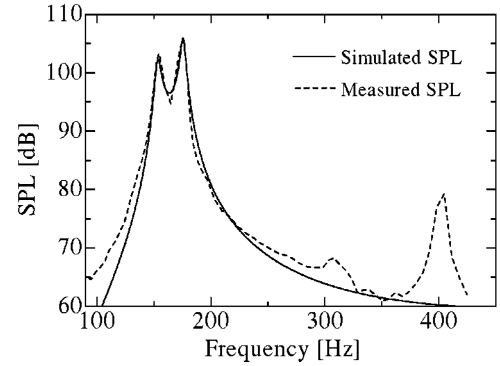

FIG. 12. Spectra of simulated and actual mokugyo.

C. Simulation

The sound spectrum of the mokugyo was simulated based on Eq. (7). Some of the parameter values, however, may be different from the actual values, since the shapes of the cavity and the side port assumed in the proposed model differ from the actual ones. W_c may also vary when struck by the drumstick. In this simulation, therefore, the additional mass of the vibratory board, m_{ad}, the stiffness of the cavity, s_c, and the additional mass of the side port, m_{ap}, were considered to be variables. These three parameters were optimized so as to match the simulated spectrum to the actual one.

First, the spectrum was calculated using Eq. (7) with the values of m_{ad}, s_c, and m_{ap} calculated in Sec. III B. Second, the optimal values for these three parameters were found by varying the values repeatedly. The convergence criteria were that the simulation error was within 1% for F_1, F_2, and the spectrum level difference at these frequencies. Here, $F_1 = 155$ Hz, $F_2 = 177$ Hz, and the level difference was 2.72 dB for the original sound spectrum as shown in Fig. 2(b).

Figure 12 shows the result obtained through the procedure described above. The final characteristics given by this simulation indicate that $F_1 = 155$ Hz, $F_2 = 177$ Hz, and the spectrum level difference is 2.72 dB. The result shows a correspondence between the simulated spectrum and the original one. The convergence values of the three parameters are $m_{ad} = 0.616$ kg, $s_c = 1.0 \times 10^5$ N/m, and $m_{ap} = 2.36 \times 10^{-3}$ kg.

Using these convergence values, the eigenfrequency of the mechanical system f_m is $1/2\pi\sqrt{s_m/m_m} = 167.5$ Hz, and that of the acoustic system f_c is $1/2\pi\sqrt{s_c/m_1} = 165.9$ Hz. These two frequencies are approximately equal. These results suggest that the mechanical vibration of the body and the acoustic vibration of the Helmholtz resonator are robustly coupled with each other, and that the beat of the waveforms shown in Fig. 2(b) is due to the mutual interaction of these two vibration systems.

From these results, the body, cavity, and side port create a structure for producing the two close resonance frequencies F_1 and F_2. We assume that these two frequencies are the most important for the timbre of the mokugyo.

IV. COMPARISON OF THEORY AND EXPERIMENT

In the final tuning process of the mokugyo, an artisan chisels three areas called “innermost,” “side port,” and

TABLE III. Measured and estimated F_1 and F_2 before and after chiseling.

Part chiseled		"innermost"		"side port"		"front"	
Weight of sawdust		227.6 g		7.29 g		205.5 g	
		F_1 (Hz)	F_2 (Hz)	F_1 (Hz)	F_2 (Hz)	F_1 (Hz)	F_2 (Hz)
Measured	before chiseling	155.0	177.0	154.1	175.4	153.1	174.5
	after chiseling	154.1	175.4	153.1	174.5	153.4	175.1
Simulated	before chiseling	155.0	177.0	154.1	176.1	153.0	175.4
	after chiseling	154.1	176.1	153.0	175.4	153.4	176.0
Relative error		−0.03%	−0.38%	0.06%	−0.52%	0.05%	−0.52%
Main factors		$\Delta s_c = -2.08\times10^3$ N/m		$\Delta m_p = +1.49\times10^{-5}$ kg		$\Delta m_d = -6.90\times10^{-2}$ kg	
				$\Delta m_{ap} = +7.5\times10^{-5}$ kg			

"front" shown in Fig. 1. In this experiment, these areas were chiseled by an artisan and the sound spectra were measured before and after chiseling. The accuracy of the simulation with the proposed model described in Sec. III was examined using the results of the measurements during the actual chiseling.

A. Experiment and simulation conditions

1. Measurement experiment method

A mokugyo with a diameter of 37 cm was used for this experiment. The material of the body was camphorwood and its density was 0.16 g/cm^3. The experimental procedure was as follows:

(a) The sound spectrum of the mokugyo was measured by the same method as described in Sec. II A.
(b) The "innermost" was chiseled by an artisan until he felt that the timbre changed.
(c) The mass of wood chips chiseled by the artisan and the sound spectrum for the chiseled mokugyo were measured.
(d) Procedures (a)–(c) were carried out for the "side port" and "front."

2. Simulation method

The difference between the volume before and after chiseling the "innermost," $\Delta W_{innermost}$, can be obtained from the mass of the wood chips and the density of the wood. $\Delta W_{innermost}$ is equal to the increase in volume of the cavity, ΔW_c, and the parameter corresponding to ΔW_c in the proposed model is the stiffness of the cavity s_c. Thus, substituting $\Delta W_{innermost}$ into ΔW_c in Eq. (12) gives the variation in stiffness, Δs_c. The simulated value of F_1 and F_2 can be obtained by substituting Δs_c into Eq. (7).

The decrease in volume of the "side port," $\Delta W_{side\ port}$, can be also obtained from the mass of the wood chips and the density of the wood. $\Delta W_{side\ port}$ is equal to the increase in the volume of the side port, W_p. Thus, the variation of air mass in the side port, Δm_p, can be written as $\Delta W_{side\ port} \times \rho_0$. In addition, the additional mass of the side port, m_{ap}, increases since the cross section S_p of the side port increases due to the chiseling. Assuming that the length of the cylinder of the port l_p remains the same, the increase in the radius of the port, Δa_p, can be calculated, and then the additional mass, Δm_{ap}, can be obtained by using Eq. (9).

Considering the structure of the mokugyo, the mass of the "front" may vary with the decrease in the equivalent mass, Δm_d, in the mechanical system. It would be difficult, however, to estimate a relationship between Δm_d and the mass of the wood chips from the "front." Thus, in this simulation, Δm_d is assumed to be a variable parameter whose maximum value is equal to the mass of wood chips. The simulated values of F_1 and F_2 are calculated using Eq. (7). The optimal value of Δm_d is found using the same method as described in Sec. III C.

B. Results and discussion

Table III shows the weight of wood chips, measured and simulated resonance frequencies F_1 and F_2 before and after the chiseling, the relative error, and the main parameters thought to affect F_1 and F_2. For "innermost" and "side port," the measured F_1 and F_2 decreased after the chiseling, and the simulated values matched the measured ones well since the relative error was within 0.52%. The chiseling of the "side port" affects F_1 and F_2 significantly in spite of the fact that the mass of the chiseled wood chips, 7.6 g, is only 0.18% of the whole mass of the mokugyo, 4.3 kg. Such significant variations in F_1 and F_2 are due to the variation of the eigenfrequency of the Helmholtz system, which varies according to the variation of the volume of the "side port," $\Delta W_{side\ port}$, and the mutual interaction of the vibration of the body and the Helmholtz system. In contrast, chiseling the "front" increased F_1 and F_2, but the simulated values still matched the measured ones well since the relative error was within 0.52%. This seems to indicate that the top and bottom portions of the instrument are moving like the tines of a tuning fork or like portions of a slot drum. The vibrational mode of the wooden shell was not changed at its lowest sound peaks. The optimal value of Δm_d was found to be -6.90×10^{-2} kg, which is about $\frac{1}{3}$ of the mass of the wood chips. The reason why Δm_d becomes $\frac{1}{3}$ of the mass of the wood chips is a topic for further research. These results suggest that the proposed model can estimate ΔF_1 and ΔF_2 using the mass of the wood chips from the three areas selected here.

V. TIMBRE EVALUATION

As described in Sec. II A, beats are observed in the sound waveforms for all mokugyos, and their periods are $1000/(F_2 - F_1)$ (ms). In this section, the effects of F_1 and F_2 on the timbre of four kinds of mokugyos (with 9-, 18-, 37-, and 90-cm diameters) are investigated psychoacoustically with the semantic differential (SD) method. It is desir-

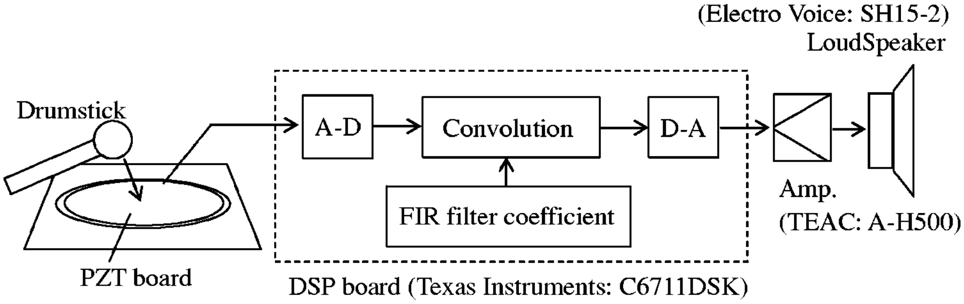

FIG. 13. Block diagram of the simulation synthesis method.

able for timbre evaluation that subjects can play the mokugyo. However, it is difficult to vary the resonance frequencies to arbitrary values by chiseling a real mokugyo. Thus, a system for simulating the sound of the mokugyo was made using a DSP board. This system can reproduce in real time simulated mokugyo sounds based on the model described in Sec. III.

A. Method of the simulation

Figure 13 shows a block diagram for synthesizing the sound using a DSP board (Texas Instruments: C6711DSK). The impact force of a drumstick was simulated as a voltage signal generated by striking a circular PZT board with a real drumstick. This signal was digitized with a sampling frequency of 2000 Hz and was transmitted to the DSP board. The parameter values of the proposed model for four kinds of real mokugyos were obtained by the same method as described in Secs. III B and C. Table IV shows sound characteristics of four real mokugyos and the values of F_1 and F_2 for each synthesized sound. Parameters in the proposed model were selected to match the F_1 and F_2 values shown in Table IV. The transfer function of the mokugyo body, i.e., FIR filter coefficients, was calculated for a sampling frequency of 2000 Hz. Then, the mokugyo sound was synthesized by convolution of the FIR coefficients with the impact force signal of the drumstick. This signal was reproduced by a loudspeaker (Electro Voice: SH15-2) via the D-A converter inside the DSP board.

B. Experiments

The timbre evaluation experiment based on the SD method was conducted in an anechoic room with a volume of 60 m^3. Subjects reproduced each sound source with the system described in Sec. V A. The process was repeated until each subject felt that the evaluation was finished. To evaluate the four real mokugyos, the real mokugyos were also struck with a drumstick by the subject. The order of the experiments was randomized. The subjects were ten males and two females with normal hearing. In the experiment, ten rating words for a bipolar scale such as ‘‘bright’’ or ‘‘soft,’’ etc., and six rating words for a monopolar scale such as ‘‘like a wooden drum’’ or ‘‘comfortable,’’ etc., were used. The bipolar scale consists of nine points: −4, Extremely; −3, Very; −2, ‘‘Dark’’ or ‘‘Hard,’’ etc.; −1, A little; 0, Midway; +1, A little; +2, ‘‘Bright’’ or ‘‘Soft,’’ etc.; +3, Very; +4, Extremely. The monopolar scale consists of seven points: 1, Not at all; 2, Not; 3, Not too; 4, A little; 5, Good; 6, Very; 7, Extremely.

TABLE IV. Sound characteristics of four real mokugyos and the values of F_1 and F_2 for the synthesized sounds.

Diameter of mokugyo (cm)	F_1 (Hz)	F_2 (Hz)	Center frequency F_c (Hz)	Values of $\Delta F = F_2 - F_1$ for synthesized sounds
9	984	1098	1041	0–200Hz(in50-Hzsteps)
18	392	427	409.5	0–60Hz(in10-Hzsteps)
37	155	177	166	0–40Hz(in10-Hzsteps)
90	70	80	75	0–20Hz(in 5-Hzsteps)

C. Results and discussion

Figure 14 shows the results of the timbre evaluation using the SD method for the real and the synthesized sounds. In this paper, only results for the mokugyo with a 37-cm diameter are shown. As shown in Figs. 14(a) and (b), the timbre of the real sound is mainly characterized by the rating words: ‘‘smooth,’’ ‘‘warm,’’ ‘‘gracious,’’ ‘‘rich,’’ ‘‘like a wooden drum,’’ and ‘‘comfortable.’’ The results for the synthesized sound with $\Delta F = 20$ Hz, which is closest to that of the real sound ($\Delta F = 22$ Hz), are similar to those for the real sound, that is, the timbre of the synthesized sound is close to that of the real sound. The six words mentioned above and ‘‘distinct’’ show higher scores. Here, $F_2/F_1 = 1.13$. On the other hand, scores for the words ‘‘distinct,’’ ‘‘clear,’’ ‘‘bright,’’ ‘‘cold,’’ and ‘‘unpleasant’’ are significantly higher for the synthesized sound with $\Delta F = 0$ Hz, and the shape of its polygon is significantly different from the real sound. The polygon of the sound for $\Delta F = 40$ Hz also differs from the others.

Figure 15 shows the relationship between F_2/F_1 and the average scores for the two monopolar rating words: (a) ‘‘like a wooden drum’’ and (b) ‘‘comfortable.’’ The results show that each evaluation score varies according to the value of F_2/F_1. As shown in Fig. 15(a), the maximum scores for ‘‘like a wooden drum’’ are 3.25, 4.41, 4.42, and 3.29. Furthermore, the rates of change from the minimum score to the maximum one are 0.24, 0.54, 0.65, and 0.15 in order of increasing mokugyo size. Thus, the maximum score and the rate of change for 18- and 37-cm mokugyos are higher than those for 9- and 90-cm mokugyos. The values of F_2/F_1 for the maximum scores are 1.16, 1.08, 1.13, and 1.17. These values are close to the slope (1.15) of the regression line described in Fig. 3. The evaluation score of the real sound of the mokugyo with a 37-cm diameter is 4.25, whereas that of the synthesized sound is 4.42. As shown in Fig. 15(b), the

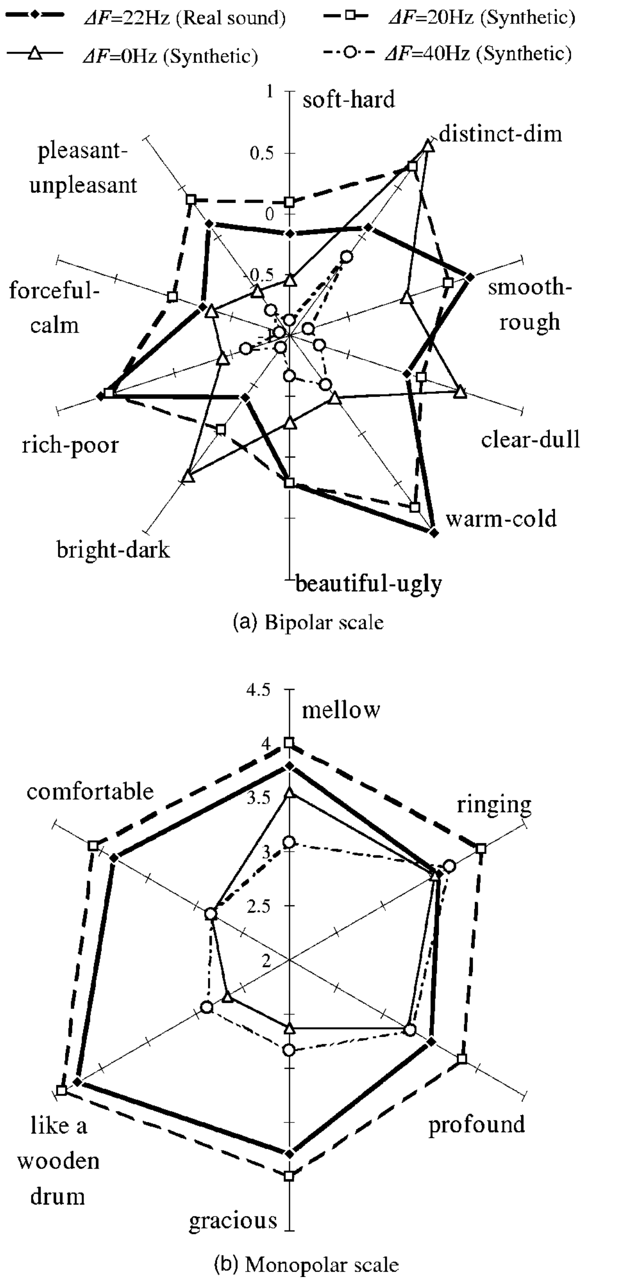

FIG. 14. Timbre evaluation results based on the SD method for a 37-cm mokugyo. (a) Results for a bipolar scale. (b) Results for a monopolar scale.

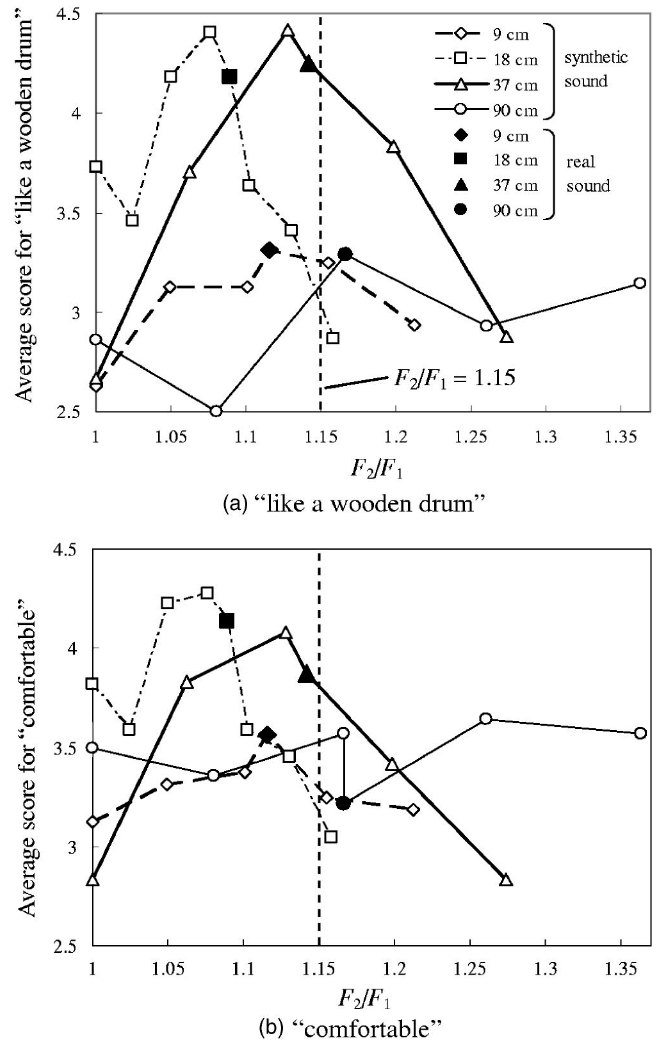

FIG. 15. Relationship between the evaluation score and F_2/F_1: (a) "like a wooden drum" and (b) "comfortable."

maximum scores for "comfortable" are 3.38, 4.27, 4.08, and 3.64 in order of increasing mokugyo size. The rates of change from minimum score to maximum one are 0.08, 0.40, 0.44, and 0.09 in order of increasing mokugyo size. Thus, the maximum score and the rate of change for 18- and 37-cm mokugyos are higher than those for 9- and 90-cm mokugyos. The values of F_2/F_1 for the maximum scores are 1.16, 1.08, 1.13, and 1.27. These values are also close to the slope 1.15. These results support our hypothesis that F_1 and F_2 are the most important factors affecting the timbre of a mokugyo. Also, the simulated sound based on the proposed physical model using a DSP board is effective.

VI. CONCLUSIONS

A traditional Japanese instrument, the mokugyo, which is made depending only on an artisan's intuition, is investigated experimentally, theoretically, and psychoacoustically from an acoustic engineering standpoint. First, the physical characteristics of the mokugyo and the drumstick were experimentally measured and analyzed, and the following results were obtained:

(a) The sound of the mokugyo involves two close spectral peaks, F_1 and F_2, and F_2/F_1 was approximately 1.15.
(b) The vibration of the driving point of the body shows a beat whose period is equal to the radiated sound wave.
(c) The duration of the impact force of the drumstick varies from 1 to 6 ms according to the material wrapped around the tip.
(d) The directivity of the F_1 and F_2 components is omnidirectional.
(e) The vibration mode of the F_1 and F_2 components is a point sound source.

Second, a physical model of the mokugyo was proposed. Because this model involves three variables which are difficult to measure, these variables are estimated using the real sound of the mokugyo. The proposed model was evaluated using a real mokugyo in the final tuning process. The results showed that the model can estimate the difference between resonance frequencies before and after chiseling from the mass of wood chips from three areas: ''innermost,'' ''side port,'' and ''front.''

Finally, the effects of F_1 and F_2 on the timbre of the mokugyo were investigated psychoacoustically using the semantic differential method. The results showed that when F_2/F_1 is approximately 1.15, the evaluation scores for the rating word, ''like a wooden drum'' or ''comfortable,'' showed higher scores.

It can be said that a mokugyo whose timbre is evaluated as a favorable sound can be made by controlling the physical characteristics corresponding to the timbre using the proposed model.

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