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Coding and Modulation Tradeoffs for Limiter-Discriminator based CPM Transceivers in a Rayleigh Fading Channel

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SUMMARY Coding and modulation tradeoffs for limiter-discriminator based CPM transceivers are examined in Rayleigh, fast fading environments. Comparisons are made on the basis of a fixed bandwidth to information rate ratio, so coded schemes and uncoded schemes can be compared fairly. It is shown that using the proper balance of modulation and coding is important to achieve good performance. It is found that combining bandwidth efficient modulation with convolutional coding achieves better performance than trellis coded modulation (TCM). The increase in performance as the code complexity is increased is also found to be larger for convolutional coding than for TCM.

key words: *CPM, limiter-discriminator, Rayleigh fading, coding*

1. Introduction

Limiter-Discriminator (L-D) detection is a simple, robust demodulation technique that has received much interest, e.g., [1]-[6]. This technique has the advantage of being very robust, which is necessary in fading environments, and very simple, which makes it attractive for portable and hand-held devices.

Portable communication systems all suffer from the effects of fading. In order to improve the performance of communication systems, channel coding techniques are a necessity. In previous work on L-D detection, the coding aspect has not been examined, so it is not clear how coding techniques affect the performance of L-D based systems.

Another problem that portable communication systems face is the limited frequency spectrum. Conventional coding techniques involve a decrease in the information rate or equivalently an increase in the bandwidth required to transmit the signal. Thus, it is desirable to use a coding technique that does not require an increase in the required bandwidth.

In this paper, we examine coding for Continuous Phase Modulation (CPM) transceivers that use a L-D detector in fast, Rayleigh fading environments. The coded scheme has the same bandwidth to information rate ratio as the uncoded scheme, so an increase in the required transmission bandwidth is not necessary. This allows us to also examine the tradeoffs involved between the amount of bandwidth used for modulation and the amount used for coding. We restrict our attention to CPM because of the properties of CPM which make it a good choice for communication over fading channels: bandwidth efficiency and constant envelope.

2. Transceiver Structure

The communication system that is used in this paper is shown in Fig. 1. A data source produces equiprobable symbols, $a_k \in \{-1, 1\}$, every T_0 seconds. These symbols are fed into a rate r convolutional encoder, which produces encoded symbols, b_k, every T seconds. In the actual implementation, the b_k are binary, but when a multilevel modulation scheme is used, the bits are grouped to form multilevel symbols, e.g., when 4-level FSK is used, the bits are taken two at a time to form quaternary symbols. Interleaving is not used, because we are interested in examining the interaction between modulation and coding only.

The encoded symbols are then sent to a CPM modulator. The output of the modulator is given by

$$s(t) = A\cos\left(2\pi h \sum_k b_k \int_{kT}^{t} g(\tau - kT)d\tau\right), \quad (1)$$

where h is the modulation index and $g(t)$ is the frequency pulse, which is zero outside the interval $[0, LT]$

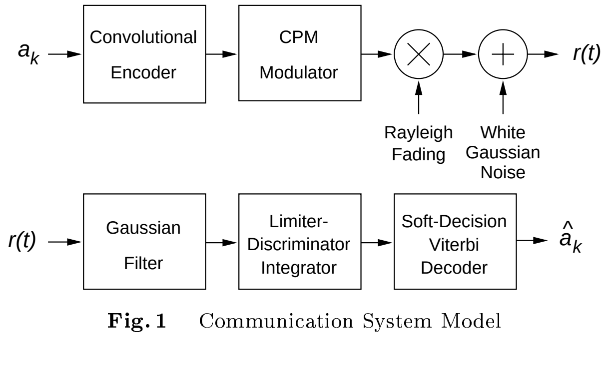

Fig. 1 Communication System Model

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Table 1 CPM Modulation Schemes

Modulation	h	L	b_k	$g(t)$
Binary FSK	1	1	$\{-1,1\}$	$1/(2T)$
4-level FSK	1/3	1	$\{-3,-1,1,3\}$	$1/(2T)$
MSK	0.5	1	$\{-1,1\}$	$1/(2T)$
GMSK ($B_b = 0.5$)	0.5	3	$\{-1,1\}$	$K\{\mathrm{erf}[cB_b(t+\frac{T}{2})] - \mathrm{erf}[cB_b(t-\frac{T}{2})]\}$, $c = \pi\sqrt{\frac{2}{\ln 2}}$

and is normalized so that its area is equal to 1/2. The parameter L is the memory length of the modulation. CPM terminology and notation in this paper follow [7]. In this paper, we consider four types of CPM: GMSK, MSK, FSK and 4-level FSK. CPM signals can be completely defined by h, L and $g(t)$. In Table 1, we list these parameters for the modulation schemes used in this paper. Note that quaternary symbols are required for 4-level FSK.

A rough comparison of the bandwidth efficiency of the FSK and MSK modulation schemes can be done by comparing the modulation index. Since the frequency pulse is the same for binary FSK, 4-level FSK and MSK, the modulation index multiplied by the largest value of b_k is the largest frequency deviation of the modulation scheme. This gives a rough bandwidth measure. From Table 1, we can see that using this bandwidth measure, the binary FSK and 4-level FSK have the same bandwidth. Compared to these two schemes, MSK has half the bandwidth. As for GMSK, the 99.9% power bandwidth, i.e., the bandwidth in which 99.9% of the signal power is contained, is roughly half that of MSK [8].

The channel model used in this paper is the standard flat, Rayleigh fading model with additive white Gaussian noise (AWGN) [9]. The fading bandwidth of the channel is given by B_f, while the AWGN has a white power spectral density equal to N_0.

Since the amplitude of the received signal varies due to the fading, the signal-to-noise ratio is given by [10].

$$\overline{SNR} = \langle |fading|^2 \rangle E_b/N_0, \tag{2}$$

where $\langle |fading|^2 \rangle$ is the average value of the squared magnitude of the fading signal and E_b is the energy required to transmit each bit.

The receiver begins by removing out-of-band interference with a pre-detection filter. In this paper, we use a Gaussian filter with a $3dB$ bandwidth denoted by B. This type of filter was also used in our previous work on limiter-discriminator detection [11]. Next, a L-D detector converts the modulated signal to baseband. The limiter part of the detector removes any envelope variations in the signal so that the discriminator output is only a function of the signal phase. The discriminator part of the detector ideally outputs the derivative of the the signal phase, i.e., the instantaneous frequency. Therefore, the response to a faded signal such as $\rho(t)\cos(2\pi f_c t + \theta(t))$ is $d\theta(t)/dt$, where $\rho(t)$ is the time varying envelope and $\theta(t)$ is the signal phase.

An integrator integrates the baseband signal for T seconds, samples the result and then resets the integrator. We assume that the receiver is perfectly synchronized so that the sample times are on the symbol boundaries. Finally, the samples are passed to a soft-decision Viterbi decoder, which produces estimates, $\hat{a}_k$, of the input symbols, a_k.

3. Coding Methods

In this paper, we consider two ways of coding the input data: Trellis Coded Modulation (TCM) and convolutional coding. To implement TCM in an non-coherent system, such as is considered in this paper, requires the use of multi-level signalling, e.g., 4-level FSK. Unfortunately, the signal distortion due to fading tends to make multi-level decisions less reliable than binary decisions. In addition, the modulation index must be reduced so that the required bandwidth is the same as the uncoded system, i.e., the transmitted frequencies must be made closer together than for the uncoded binary system.

The convolutional coding approach is as follows. Compared to a benchmark, we use a more bandwidth efficient modulation scheme in conjunction with a rate r convolutional code and an pre-detection filter bandwidth of $B{\cdot}r$. Since the information rate is decreased by using a convolutional code, the ratio of the bandwidth to the information rate is the same as the benchmark scheme.

In this paper, we consider binary FSK ($h = 1.0$) as a benchmark and a pre-detection filter bandwidth of $BT = 1$. For the bandwidth efficient modulation schemes, MSK and GMSK are used. The convolutional codes that we use are taken from pages 330-331 of Lin & Costello [13]. These codes have the maximum d_{free} for a given rate and number of encoder states.

We will also compare the performance of convolutionally coded CPM with TCM. The TCM codes that are considered are taken from [12]. These codes are combined with 4-level continuous phase FSK modulation with a modulation index $h = 1/3$. Phase modulation cannot be used with L-D detection because of the noncoherent nature of the demodulation process.

4. Performance

We simulated the performance of coded CPM for several fading rates. The results for $B_f T = 0.01$ are shown in Figs. 2 and 3 for MSK and GMSK respectively. The probability of bit error is denoted by P_e. Error floors are present in the results because of inter-symbol interference that results from the pre-detection filter. Similar results were also obtained for other fading rates, but

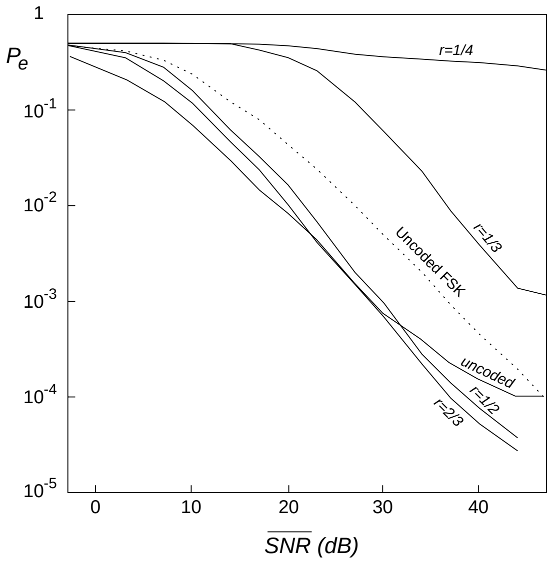

Fig. 2 Performance of coded MSK for $B_f T = 0.01$ using 4-state codes. r is the code rate.

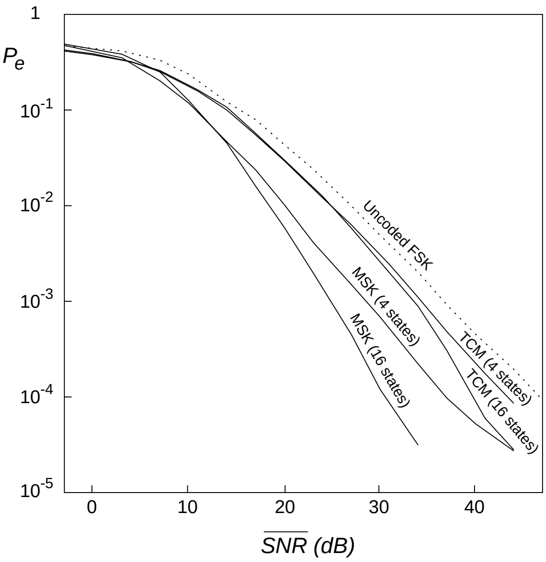

Fig. 4 Performance comparison of MSK ($r = 2/3$) and TCM using 4 and 16 state encoders for $B_f T = 0.01$.

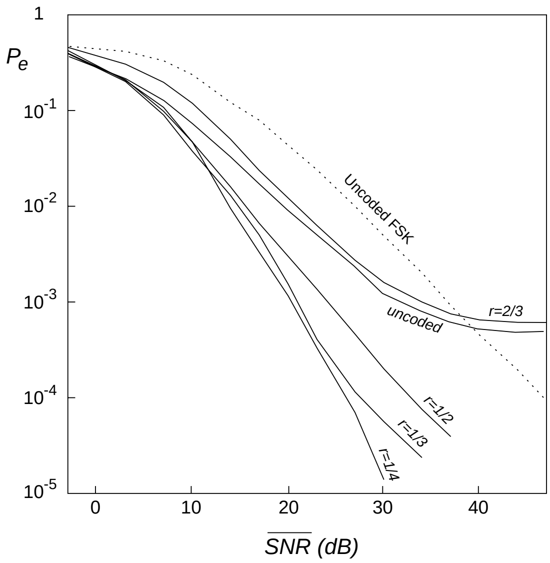

Fig. 3 Performance of coded GMSK for $B_f T = 0.01$ using 4-state codes. r is the code rate.

are not shown here due to space limitations.

The results for MSK show that a convolutional code rate of 2/3 gives the best performance. The error floor is better than that of uncoded FSK and the performance of coded MSK at $P_e = 10^{-3}$ is approximately $8dB$ better. As the rate and filter bandwidth decrease, the performance of coded MSK becomes worse. This is due to the fact that the performance without coding becomes worse as the bandwidth is decreased. This decrease cannot be overcome by the increase in coding gain. On the other hand, we can see that if the rate is increased further, the overall performance becomes worse.

For GMSK, however, a convolutional code rate of 1/4 gives the best performance. At $P_e = 10^{-3}$, the improvement over uncoded FSK is approximately $16dB$. For GMSK, the performance as the rate and bandwidth decrease gets better because the performance without coding is not affected as much by a decrease in filter bandwidth. This result is due to the bandwidth efficiency of GMSK.

From the above discussion, we can see that roughly 2/3 of the total bandwidth should be used for modulation and the other 1/3 of the bandwidth for coded MSK. For coded GMSK, on the other hand, only 1/4 of the total bandwidth is should be used for the modulation.

In both cases, we can see that the best code rate is roughly equal to the bandwidth efficiency of the modulation scheme relative to FSK. This can be explained as follows. The performance without coding is not affected by a decrease in the pre-detection filter bandwidth up to a certain point, which depends on the bandwidth efficiency of the modulation scheme. However, the performance of the code improves as the rate is lowered. Therefore, the net result is an improvement in performance relative to the uncoded scheme. After a certain point, the performance without coding decreases faster than the code performance improves, resulting in a net decrease in performance. Since GMSK is more bandwidth efficient than MSK, a lower rate and filter bandwidth can be used.

In Fig. 4, the performance of convolutionally coded CPM is compared to TCM for $B_f T = 0.01$. From the figure, we can see that convolutional coding results in better performance than TCM. At an error rate of 10^{-3}, coded CPM is approximately $6dB$ better. We can also see that the performance of convolutionally coded CPM improves much more than TCM when the code complexity is increased. The difference in performance can be attributed to the fact that multi-level decisions, such as those necessary in TCM, are less reliable for the L-D detection method examined in this paper.

5. Conclusions

In this paper, we have examined the tradeoffs between coding and modulation for limiter-discriminator based CPM transceivers in Rayleigh, fast fading environments. The performance of coded CPM schemes that have the same bandwidth to information rate ratio as uncoded schemes was evaluated by computer simulations. It was found that convolutional coding of CPM can give an improvement of $8dB$ to $16dB$ over uncoded FSK at an error rate of 10^{-3}. Furthermore, it was found that the percentage of the total transmission bandwidth that should be used for modulation is determined by the bandwidth efficiency of the modulation scheme.

Also, convolutionally coded CPM was compared to TCM. The performance of convolutional coding was found to be approximately $6dB$ better than that of TCM at an error rate of 10^{-3}. It was also found that the performance improvement obtained by increasing the code complexity is larger for convolutional coding than for TCM. The results indicate that convolutional coding of CPM is more suitable for limiter-discriminator based transceivers than TCM.

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