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**Technical Note**

# Single-carrier Transmission Frequency-domain Equalization Based on a Wiener Filter for Broadband Wireless Communications

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Recently, frequency-domain equalization for single-carrier transmission (SC-FDE) has been given much attention. For example, the enhanced mobile phone system, a SC-FDMA (Single-carrier frequency division multiple access) method using SC-FDE and multiple access will be adopted. However in previous research, there are many papers describing the features and advantages of SC-FDE based on a comparison of SC-FDE and orthogonal frequency-division multiplexing (OFDM) systems. In this technical note, we discuss single-carrier transmission equalization in the time-domain (SC-TDE) and SC-FDE in a unified way centered on the Wiener filter based on the minimum mean square error (MMSE) criterion. The reason to take up a Wiener Filter is that it is a basic filter based on the MMSE criterion. Also, we explain the basic principle of the SC-FDE and SC-FDMA in an organized and systematic way. Moreover, we point out the physical meaning of the Wiener solution in SC-FDE and relationship between SC-TDE and SC-FDE Wiener solutions. As a result, we show useful information and pointers, especially for when we want to replace existing SC-TDE technology with SC-FDE technology.

**Keywords** : Single-carrier time-domain equalization (SC-TDE), Single-carrier frequency-domain equalization (SC-FDE), Single-carrier frequency division multiple access (SC-FDMA), Wiener filter, Cyclic-prefix (CP)

## 1. Introduction

In the enhanced mobile phone system, one of the specifications requires that the peak transmission rate is more than 100Mbps for the down-link and more than 50Mbps for the up-link in a bandwidth of 20MHz[1]. Moreover, in next generation mobile communications, ultra high speed of about 1Gbps is required[2]. Broad-band wireless channels for high speed and mass transmission are frequency selective channels, which are composed of a lot of spread paths with different delay times. However, in such environments, inter-symbol interference (ISI) due to the effects of multi-path fading occurs, resulting in a performance loss[3].

The main techniques to counteract ISI include time-domain equalization for single-carrier transmission (SC-TDE)[4]-[6] and orthogonal frequency-division multiplexing (OFDM)[7][8] and multi-carrier code division multiple access (MC-CDMA)[9] technology for multi-carrier transmission. For SC-TDE, the decision feedback equalizer (DFE)[10][11] and the maximum likelihood sequence estimation (MLSE)[12] Equalizer for TDMA access mainly have been studied for use in mobile communications. However, in SC-TDE techniques, the computational cost to equalize depends on the paths of the delayed waves. In broadband transmission, when the resolution time improves and the number of paths that can be separated increases, computational costs increase rapidly. As for OFDM, the technology has already been adopted for high-speed wireless LAN, digital terrestrial television and mobile-WiMAX[13], and therefore has been aggressively studied. However, in OFDM transmission, the peak to average power ratio (PAPR) of the transmitted signal rises when more independent sub-carriers are used, and the efficiency of the power amplifier decreases remarkably. Therefore, it is difficult to use in portable terminals which are driven by a battery.

So, recently, frequency-domain equalization for single-carrier transmission (SC-FDE) technology has been given much attention[14]-[24]. Actually, for the up-link in the 3.9G-LTE system an SC-FDMA method using SC-FDE and multiple access are adopted[16]. Roughly speaking, equalization is done for single-carrier transmission by inserting a cyclic-prefix (CP) in the transmitter, then multiplying by a single equalization tap in the frequency domain via FFT/IFFT processing in the receiver[14][15]. So, equalization can be done using lower a computational cost than using SC-TDE. Moreover, we can get the frequency diversity effect and suppress the ISI with SC-FDE using the MMSE (minimum mean square error) criterion As a result, better BER (bit error rate) performance improvement with low computational cost can be expected in Broad-band Wireless Communications using SC-FDE. The above mentioned PAPR trouble is overcome since single-carrier transmission is used. For example, SC-TDE, which is called "rake-compound", has been used in DS-CDMA using single-carrier transmission. However though it is effective to use this method for a bandwidth of several MHz, the number of compound paths increases for bandwidths larger than this and inter-path interference (IPI) in severe frequency selective channels occurs, resulting in poorer performance[25]. On the other hand, if SC-FDE instead of rake-compound is adopted using DS-CDMA it has been shown that the performance is likely equivalent to that using multi-carrier CDMA[18].

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In (15), (17), (20) the features of the SC-FDE are described by comparing SC-FDE with OFDM, and the effectiveness of the SC-FDE is shown using computer simulations. In (19), the results of the latest research for SC-FDE are reviewed, including the derivation of the channel capacity and applications to multiple-input multiple-output (MIMO) systems. In (21), the framework for block transmission is reviewed in a unified way using the matrix expression mentioned in (22). We can understand the basic principle of both OFDM and SC-FDE transmission well by thinking of them as block transmission methods using a CP. As mentioned above, SC-FDE is being studied actively. However, there are currently many communication systems using SC-TDE[26], for example the DFE in global system for mobile communications (GSM) second generation mobile phone systems. In the future, we can image the demand to replace existing SC-TDE technology with SC-FDE technology. However, in previous research, there are few papers describing the features and advantages of SC-FDE based on a comparison of SC-TDE and SC-FDE systems.

In this technical note, we discuss single-carrier adaptive equalization using the MMSE criterion in the time-domain (TD) and the frequency-domain (FD) in a unified way centered on the Wiener Filter. In particular, we explain the basic principle of the SC-FDE in an organized and systematic way so that it can be used to help understand many other papers. Moreover, we consider the significance of the CP and the physical meaning of the Wiener solution in the frequency domain. Also, this technical note is based on the discussion in (27).

The rest of this technical note is organized as follows. In section 2, we describe the SC-TDE system. In section 3, the SC-FDE/SC-FDMA systems are considered. In section 4, we evaluate single-carrier equalization by comparing SC-TDE and SC-FDE. Finally, we present our conclusions in section 5.

Hereafter, our discussion will be in the equivalent low frequency region. Basically, it is noted that n denotes the time in the time-domain and k denotes the frequency point in the frequency-domain. Also, we call the time domain as 'TD' and the frequency domain as 'FD'.

## 2. Single-Carrier Time-Domain Equalization

A Wiener filter is used to get a suitable frequency response to a wide-sense stationary input which is a mixture of the original signals and noise signals by using the MMSE criterion between the filter output and the desired signal. Using the MMSE criterion is equivalent to suppressing the ISI to a minimum. In this section, first we describe a continuous-time system for SC-TDE. Next, it is extended to a discrete-time system.

**2.1 Continuous-time System** In Fig.1, a transmission model for this situation is shown (5). As shown in Fig.1, signals are defined and assumed to be stationary random processes. It is our goal to get suitable estimated signals $\hat{d}(t)$ for desired signals $d(t)=x(t)$ from observation signals $y(t)=x(t)+n(t)$ which are a mixture of original signals $x(t)$ and additive noise signals $n(t)$. This problem is to decide the frequency characteristic $W(f)$ to minimize the evaluation function $J$ based on the MMSE criterion. Here, m is an integer.

$$J = E\left[\{e(t)\}^2\right]. \quad \cdots (1)$$

We solve equation (2), and the desired filter frequency response $W_{time\_opt}(f)$ is given by equation (3)[5].

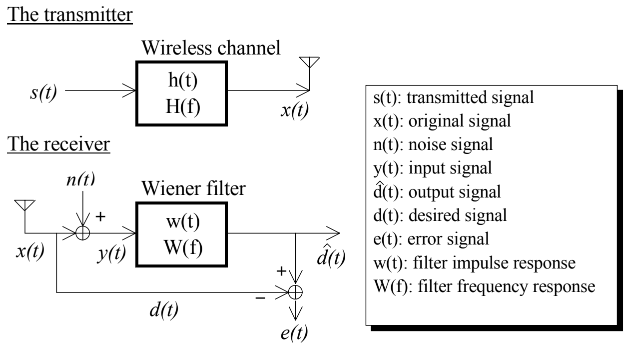


Fig. 1. Continuous-time transmission model for SC-TDE.

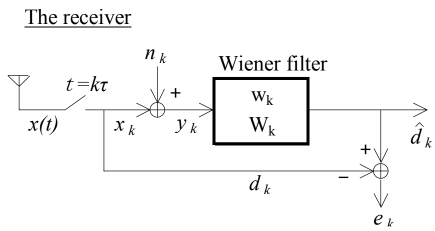


Fig. 2. Discrete-time transmission model for SC-TDE.

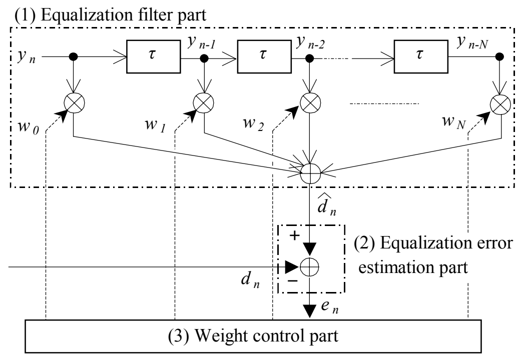


Fig. 3. SC-TDE model based on a Wiener filter.

$$\partial J / \partial w_m = 0 . \quad \cdots (2)$$

$$W_{time\_opt}(f) = \frac{S_x(f)}{S_x(f)+S_n(f)} . \quad \cdots (3)$$

Here, $S_x(f)$, $S_n(f)$ are the power spectral densities of the original signal and the noise signal respectively. Also, $w_m$ are filter coefficients. From equation (3), we can see that the Wiener filter frequency response is expressed as the ratio of the signal power to the total power. This expression (3) is necessary for the wiener solution in the frequency domain, i.e.,[24].

**2.2 Discrete-time System** We assume finite input signals and consider a discrete-time system. In this case, though a Wiener filter is constructed only at one data sampling point, this is not a problem in digital communications. In Fig.2, a transmission model for a discrete-time system is shown. The definitions of the signals are as in Fig.1. In Fig.3, a Wiener filter structure for SC-TDE is shown. The model shown in Fig.3 consists of the following three parts.

(1) Equalization filter part

The input signal $y_n$, the filter coefficients (tap gains) $w_n$ and the filter output are as follows.

Input Signal:

$$\mathbf{y}_n = \begin{bmatrix} y_n & y_{n-1} & \cdots & y_{n-N} \end{bmatrix}^{\mathrm{T}} . \quad (4)$$

Filter Coefficients:

$$\mathbf{w}_n = \begin{bmatrix} w_n & w_{n-1} & \cdots & w_{n-N} \end{bmatrix}^{\mathrm{T}} . \quad (5)$$

Output Signal:

$$\hat{d}_n = \mathbf{w}_n^T \mathbf{y}_n . \quad (6)$$

(2) Equalization error estimation part

We estimate the error signal $e_n$ from the filter output and the desired signal.

Error Signals:

$$e_n = \mathbf{d}_n - \mathbf{w}_n^T \mathbf{y}_n . \quad (7)$$

(3) Tap gain control part

We update each tap gain to minimize the variance of the equalization error. We assume that the signals $y_n$ and $d_k$ are weak stationary processes, the filter coefficient $w_{opt}$ which minimizes the evaluation function $J_d = E\left[\mathbf{e}_n^2\right]$ is as follows using the approach mentioned in the continuous-time system. Also, $w_{opt}$ is called a Wiener solution[6].

$$\mathbf{w}_{\mathbf{opt}} = \mathbf{R}^{-1}\mathbf{p} . \quad (8)$$

$$\mathbf{R} = E\left[\mathbf{y}_n \mathbf{y}_n^{\mathbf{T}}\right] . \quad (9)$$

$$\mathbf{p} = E\left[\mathbf{y}_n \mathbf{d}_n\right] . \quad (10)$$

Here, R is the auto-correlation matrix and p is the cross-correlation vector of $y_n$ and $d_n$. In section. 4, we discuss this fact in detail.

**2.3 Performance Evaluation** We easily evaluate the performance of SC-TDE with a Wiener filter using computer simulations. The simulation parameters are random data of 0 and 1 as transmitted data, QPSK modulation, coherent detection, frequency selective fading which consists of two Rayleigh waves with equal power levels, a symbol length as delay term and semi-static environment. In Fig.4, the simulation results are shown. For comparison, we show the performance without a Wiener filter.

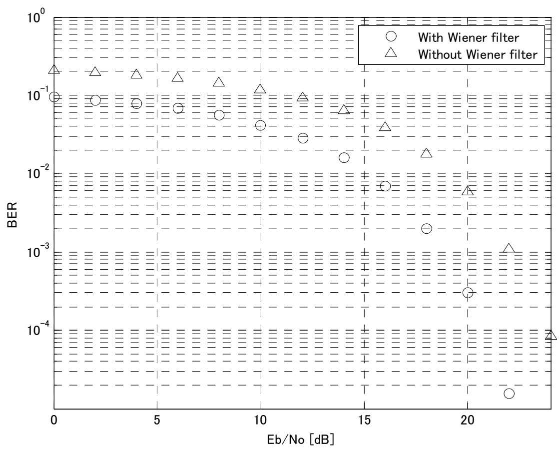


Fig. 4. BER performance for SC-TDE using a Wiener filter.

From the results, the effectiveness of a Wiener filter is shown. In particular, a BER of $10^{-4}$ appeared at values of $E_b/N_0$ greater than about 3 [dB].

## 3. Single-Carrier Frequency-Domain Equalization

**3.1 System Model** In this chapter, we discuss SC-FDE corresponding to the SC-TDE transmission model mentioned in chapter 2. To conserve space, we refer the reader to other papers, for example[19] for the simulation results. In Fig.5, we show the transmission model in the frequency domain. In this section, we mainly discuss discrete-time systems using SC-FDE. First, in the transmitter the discrete serial transmitted symbols $\{x_n\}$, with time index n, are divided into parallel streams in blocks of size N. Moreover, a CP consisting of a part of the end of the block (size: $N_g$ symbols) is inserted at the beginning of the block. The effect of the ISI due to multipath fading is eliminated by inserting the CP. We describe this point in chapter 3.2 in detail. In this technical note, we use GI (Guard Interval) to refer to the head interval of a block and CP to refer to the wave inserted into the GI. As a channel model, we assume fading channels which consist of L distinct propagation paths. Assuming the complex valued path gain is $h_l$ and the time delay of the $l$-th path is $\tau_l$, the impulse response of the channel is given by equation (12). Here, $\delta(\tau)$ is the unit impulse function. In the receiver, the CP is eliminated. The received symbol stream of size N, $\{y_n\}$, is transformed by an N point discrete Fourier transform (DFT) into the FD signal $\{Y_K\}$. Equalization can be done in the FD with a block whose size is N. In the TD and FD, the transmitted signal, the channel model, and the received signal are as follows.

The transmitted signal:

$$\begin{aligned} x_n &= \frac{1}{N}\sum_{k=0}^{N-1} X_k W_N^{-nk} \quad (\mathrm{TD}) \\ X_k &= \sum_{n=0}^{N-1} x_n W_N^{nk} \quad (\mathrm{FD}) \end{aligned} . \quad (11)$$

Here, $W_N$ is a rotation operator.

$$W_N = exp\left(-j\frac{2\pi}{N}\right)$$

The channel model:

$$\begin{aligned} h_n &= \sum_{l=0}^{L-1} h_l \delta(n-\tau_l) \quad (\mathrm{TD}) \\ H_k &= \sum_{l=0}^{L-1} h_l W_N^{n\tau_l} \quad (\mathrm{FD}) \end{aligned} . \quad (12)$$

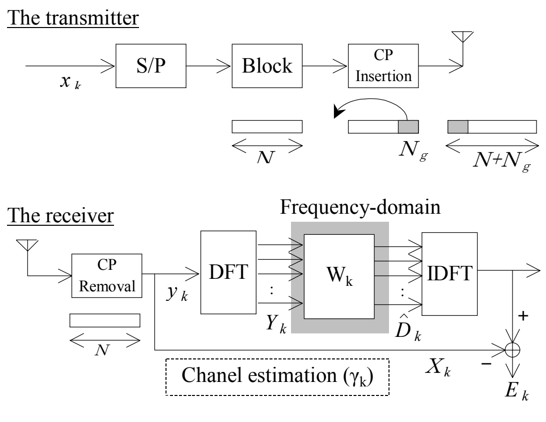


Fig. 5. Discrete-time transmission model for SC-FDE.

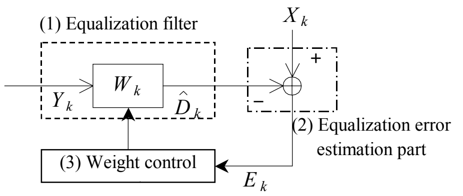


Fig. 6. SC-FDE model based on a Wiener filter.

The received signal:

$$\begin{aligned} y_n &= \sum_{l=0}^{L-1} h_l x_{n-l} + n_n \quad \text{(TD)} \\ Y_k &= H_k X_k + N_k \quad \text{(FD)} \end{aligned} \quad \text{......} (13)$$

Hereafter, we describe the process of adaptive equalization in the FD in detail. In Fig.6, a Wiener filter structure for SC-FDE is shown. As well as TD shown in Fig.3, we consider the following three parts.

(1) Equalization filter part

The filter output, i.e. the equalized wave $\hat{D}_k$, is as follows using the received signal $Y_k$ and the weight $W_k$ in the FD.

$$\begin{aligned} \hat{D}_k &= W_k Y_k \quad \text{(FD)} \\ \hat{d}_n &= \frac{1}{N}\sum_{k=0}^{N-1} \hat{D}_k W_N^{-kn} \quad \text{(TD)} \end{aligned} \quad \text{......} (14)$$

(2) Equalization error estimation part

The error signal is as follows using the expression of the TD.

$$\begin{aligned} e_n &= \hat{d}_n - d_n \\ &= \frac{1}{N}\sum_{k=0}^{N-1} (W_k Y_k - X_k) W_N^{-kn} \quad \text{(TD)} \\ E_k &= \sum_{n=0}^{N-1} e_n W_N^{kn} \quad \text{(FD)} \end{aligned} \quad \text{......} (15)$$

(3) Tap gain control part

In the FD as well as the TD, we calculated suitable tap gains $W_{freq\_opt}$ by using the MMSE criterion. As well as a continuous-time system, and the optimal tap gain vector $W_{freq\_opt}$ is as follows. (See Appendix 1).

$$W_{freq\_opt} = \frac{H_k^*}{|H_k|^2 + 1/\gamma_k} \quad \text{(FD)}. \quad \text{......} (16)$$

Here* denotes complex conjugation and $\gamma_k$ is the signal to noise ratio. In practice, we must estimate $\gamma_k$ in some way. However, here, it is assumed that we estimate it perfectly. This result is similar to the result shown in (19) and we could check the validity of our approach shown in this paper. Also, the filter output and the MMSE of the error signal are as follows.

$$\hat{D}_{opt} = \frac{|H_k|^2}{|H_k|^2 + 1/\gamma_k} X_k + \frac{H_k^* N_k}{|H_k|^2 + 1/\gamma_k} \quad \text{(FD)}. \quad \text{......} (17)$$

$$E\left[|E_{opt}|^2\right] = \frac{1}{N}\sum_{k=0}^{N} \frac{1/\gamma_k}{|H_k|^2 + 1/\gamma_k} \frac{E\left[|X_k|^2\right]}{N} \quad \text{(FD)}. \quad \text{......} (18)$$

From equation (17), we can check the fact that when the value of $\gamma_k$ is large, the signal element $X_k$ dominates. On the other hand, when the value of $\gamma_k$ is small, the noise element $N_k$ dominates.

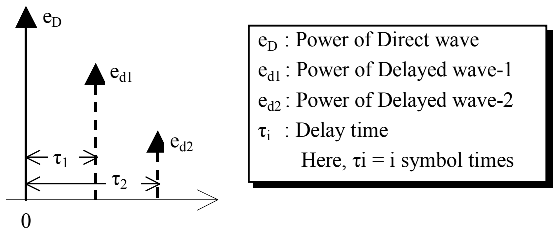


Fig. 7. A sample of the delay profile.

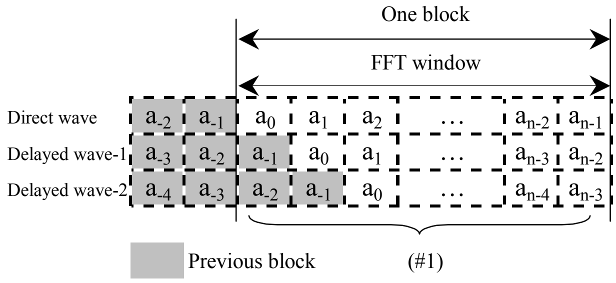


Fig. 8. The case where CP has not been inserted.

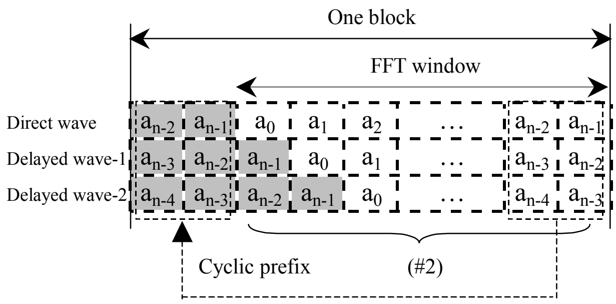


Fig. 9. The case where CP has been inserted.

From equation (18), we can check the fact that when the value of $\gamma_k$ is large, we can get a smaller equalization error.

### 3.2 Meaning of the Cyclic-prefix

(1) The update unit of the wiener solution

We assume a situation when a direct wave and two Rayleigh waves, whose delays are a symbol time, arrive. In Fig.7, a sample of the delay profile is shown. In this situation, we consider the meaning of the CP. First, we consider the case where CP has not been inserted, shown in Fig.8. In Fig.8, the signal of the previous symbol is mixed in one block period. As a result, ISI occurs. In other words, the signal in one block period is not periodic (see #1 in Fig.8) and the signal spectrum is continuous. In this case, whenever the time of one symbol length advances we must define a new block and solve the wiener solution. Oppositely, if we can consider a one block signal to be a periodic signal, the signal spectrum is discrete. So, we solve the wiener solution for the spectrum. Moreover the value in one block is common, and the update of the wiener solution is done not symbol by symbol but block by block. The CP is the method where signals in one block are considered to be periodic.

Next, we consider the case where CP has been inserted, shown in Fig.9. In the following explanation, we assume the situation that the length of the CP is smaller than the maximum time of the

delay waves. In Fig.9, it is shown that parts. By adding CP the delayed signals in the block head parts are the signals in the same block and not from the previous block. Then, the signals in one block are considered to be pseudo-periodical objects (see #2 in Fig.9). So, the signal spectrum is discrete and we can use the FFT (fast Fourier transform). As a result, as we mentioned above the update of the wiener solution can be done block by block. the signals in the block end parts are copied onto the block head However, in the receiver the data remaining after the CP is removed is demodulated (refer to the FFT interval in Fig.9), so the transmission efficiency is degraded by inserting the CP. Also, the BER performance is degraded when the length of the CP is longer than the maximum time of the delay waves[17].
Therefore, the length of the CP is decided totally from the viewpoint of the delay times of the delay waves, the transmission efficiency, and so on.

(2) Eigenvalue decomposition of circulant matrix

Next, we consider the meaning of the CP mathematically using a matrix. The signal vector after inserting the CP, $y$, is given by the product of the channel impulse matrix H and the transmitted signal vector a.

$$\mathbf{y} = \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix} = \begin{pmatrix} h_0 & h_{N-1} & \cdots & h_1 \\ h_1 & h_0 & \vdots & h_2 \\ \vdots & \vdots & h_0 & \vdots \\ h_{N-1} & h_{N-2} & \cdots & h_0 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ \vdots \\ a_{n-1} \end{pmatrix} = \begin{pmatrix} \mathbf{h_0} & \mathbf{h_1} & \cdots & \mathbf{h_{n-1}} \end{pmatrix} \mathbf{a} = \mathbf{Ha} \quad (19)$$

Here, $h_0$ is the channel impulse response vector and the first row of matrix H. The elements of the second row of matrix H are shifted by one element from $h_0$, that is, the first row of matrix H. Similarly, the elements of the third row of matrix H are shifted by two elements from $h_0$. A matrix with this structure is called a circulant matrix[28]. We show a derivation example of a circulant matrix in Appendix 2.

Now, we consider the following DFT matrix.

$$\mathbf{F} = \frac{1}{\sqrt{N}} \begin{pmatrix} (W^0)^0 & (W^0)^1 & \cdots & (W^0)^{N-1} \\ (W^1)^0 & (W^1)^1 & \vdots & (W^1)^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ (W^{N-1})^0 & (W^{N-1})^1 & \cdots & (W^{N-1})^{N-1} \end{pmatrix} . \quad (20)$$

Here, $W$ is a rotation operator. When the DFT matrix is a Unitary matrix, Eigenvalue decomposition of matrix H is done using matrix F.

$$\mathbf{H} = \mathbf{F}^H \mathbf{\Xi} \mathbf{F} . \quad (21)$$

$$\mathbf{\Xi} = \begin{pmatrix} H_0 & & & \mathbf{0} \\ & H_1 & & \\ & & \ddots & \\ \mathbf{0} & & & H_{N-1} \end{pmatrix} . \quad (22)$$

Here, the eigenvalues of matrix $\mathbf{\Xi}$ are the DFT of the impulse response, that is, the frequency response of the channels. When expression (21) can be used, equalization in the receiver can be very efficiently done. In other words, as shown in Fig.10, the

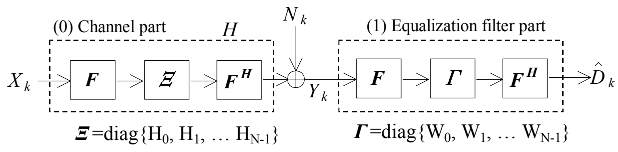


Fig. 10. SC-FDE structure using a circulant matrix.

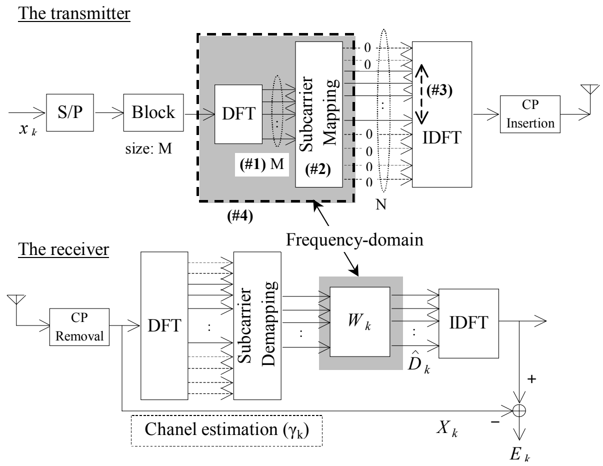


Fig. 11. SC-FDMA model.

processing of the equalization filter part corresponds to transforming the received signal $X_k$ using the DFT, multiplying $X_k$ and $W_k$ in every frequency component and converting the result back into the time domain using the IDFT. When the MMSE criterion is used, $W_k$ is expressed by (16). $F$ and $F^H$ can be calculated with lower computational costs using the FFT and IFFT. Since $\Gamma$ is a diagonal matrix, the computational cost is N multiplications. Therefore, the meaning of the CP is as follows. The channel matrix created by inserting and removing a CP is a circulant matrix and can be diagonalized using the DFT matrix.

As a result, equalization in the receiver can be very efficiently done. Also, when the length of the CP is longer than the maximum time of the delay waves, some element of the circulant matrix is missed. The ISI measure techniques are proposed for such cases[23].

**3.3 SC-FDMA (DFT-Spread OFDM)** A SC-FDE basic system is shown in Fig.5. In this chapter, we explain the application of the system shown in Fig.11, that is, SC-FDMA. One block that consists of M modulated symbols is transformed by an M point DFT after S/P conversion. The input streams of size N (here, M<N) to the IDFT are as follows. The streams from the DFT outputs mapped to the user's frequency bands and the zero streams mapped to the other frequency bands. Then, the output of the IDFT is a single-carrier signal. The meaning of "SC-FDMA" is as follows[17].

- SC (Single-carrier): Sequential transmission of the symbols over a single frequency carrier.
- FDMA: User multiplexing in the frequency domain. Here, we consider the four parts (#1-4) shown in Fig.11.

**(#1)**

When the sampling-rate of the IDFT is fs, the bandwidth of the transmitted signal BW is as follows by zero padding the IDFT inputs.

$$BW = \frac{N}{M} f_s \, . \quad \cdots\cdots (23)$$

In other words, dynamic allocation of bandwidth can be controlled by changing the DFT size M according to the bandwidth which we want to allocate. For example, it is possible to allocate a wide bandwidth to users who want to communicate with high data rates. In addition, when M is equal to N the DFT and IDFT processing completely negate each other, so this case is meaningless.

**(#2)**

In this part, mapping control of the sub-carriers is done. The main mapping methods are as follows. These features are described in (24).

- Localized FDMA

The streams from the DFT outputs are mapped continuously and locally.

- Distributed FDMA

Zeros are inserted between the DFT outputs and the resulting stream is mapped discretely.

**(#3)**

The position of the transmitted signals in the frequency-domain can be shifted by shifting the IDFT inputs which are mapped from the DFT outputs. Therefore with #1, flexible allocation of bandwidth can be controlled using SC-FDMA.

**(#4)**

As this part can be regarded as a pre-coding process based on the DFT, some people call SC-FDMA "DFT-spread OFDM".

## 4. Evaluation of Single-Carrier Equalization

**4.1 Relationship between SC-TDE and SC-FDE Wiener Solutions** We consider a Wiener solution in the frequency domain as shown in equation (16). Equation (16) is as follows using a Wiener solution, which is expressed as equation (3), derived in the time domain (See Appendix 3).

$$W_{freq\_opt}(f) = \frac{H^*(f)}{|H(f)|^2} W_{time\_opt}(f) \, . \quad \cdots\cdots (24)$$

From equation (24), we can confirm that the frequency response of the Wiener filter in the frequency domain conforms to that of the Wiener filter in the time domain. Here, we can see that $W_{time\_opt}(f)$ is expressed as the frequency response of the received filter and the term $H^*(f)/|H(f)|^2$ is compensation for the channel frequency response. In other words, only the signal element is extracted from the signal buried in the noise and the ISI caused in wireless channels is suppressed as much as possible. Also, the filter given by equation (24) can be thought of as an equalizer that compensates for the distortion caused by the receiver filter and the channel at the same time. In Fig.12, the relationship between SC-TDE and SC-FDE Wiener solutions, which are described above, is shown. Here, the definitions of the signals are as in Fig.1, and $S_x(f)$, $S_n(f)$, $S_s(f)$ is the power spectral densities of the signal $x(t)$, $n(t)$, $s(t)$.

**4.2 Comparison of SC-TDE and SC-FDE** The features of SC-TDE and SC-FDE are summarized in Table 1. In Fig.13, the image shown is a comparison of SC-TDE and SC-FDE waves. In Fig.13, the mapping for transmitted QPSK data symbols is shown for both SC-TDE and SC-FDE. Items 1-4 are the results from section 2 and 3. To conserve space, as for item 5, we do not discuss BER in detail in this paper. However, it is shown that the BER performance of SC-FDE is more excellent than that of

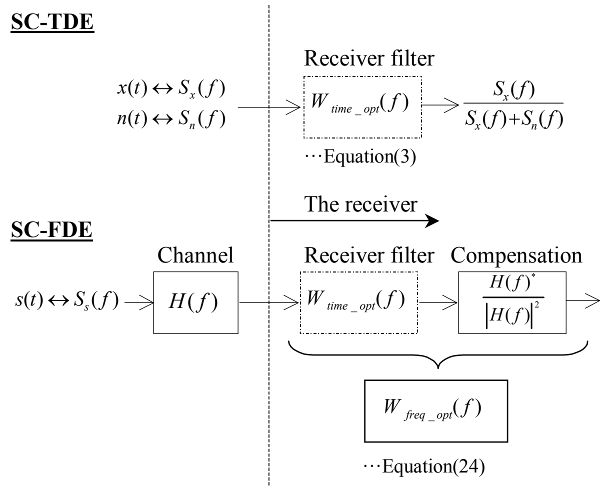


Fig. 12. Relationship between SC-TDE and SC-FDE Wiener solutions (Refer to Fig.1).

Table 1. Comparison of SC-TDE and SC-FDE.

| Item | SC-TDE | SC-FDE |
|---|---|---|
| 1. Equalization domain | Time domain | Frequency domain |
| 2. Cellular phone system generations | 2G: TDMA/DFE | 3.9G: SC-FDMA (up-link) |
| 3. Equalization (update of tap gains) step | Symbol by symbol | Block by block using DFT/IDFT |
| 3.1 Computational cost | Increases as the number of paths increases | Low |
| 3.2 Calculation of suitable tap gains | Very difficult | Very easy |
| 3.3 Tracking changes in the channel response | Comparatively, the tracking is good, and there is no influence on the equalization performance. | The equalization performance is degraded due to changes in the channel response. |
| 4. Tolerance to multi-path (ISI) | Comparatively poor | Good |
| 5. BER | Poorer than SC-FDE. | Excellent |
| 6. Amplifier compatibility | Good | |
| 7. Allocation of bandwidth | Not Flexible | Flexible |
| 8. OFDM compatibility | Not good | Very good |

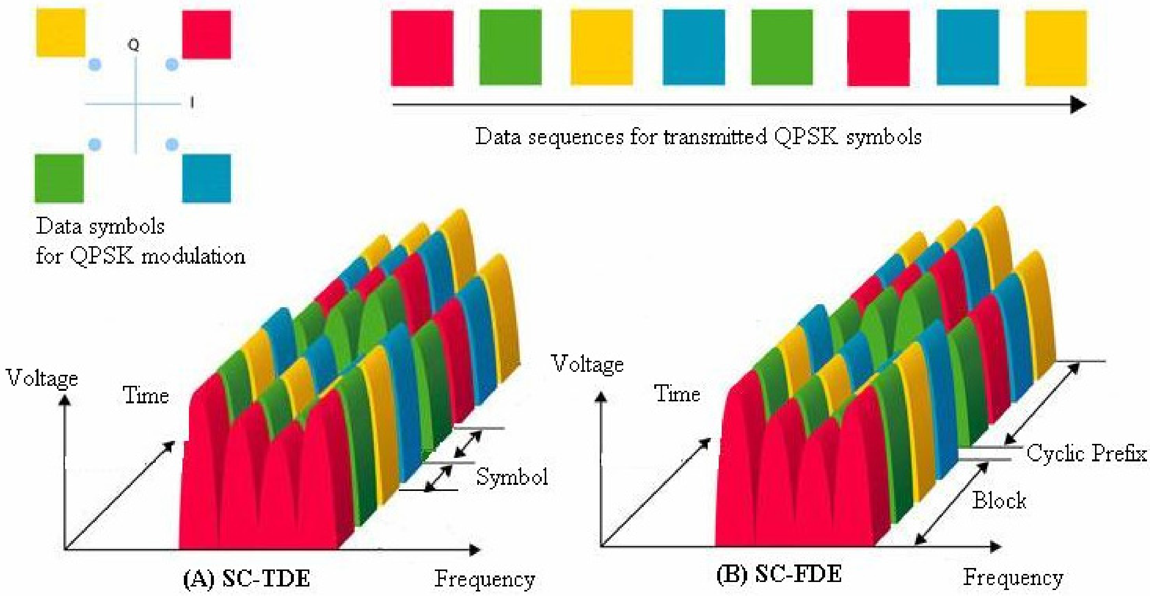


Fig. 13. Image of SC-TDE and SC-FDE Waves (some parts of (16) are modified).

SC-TDE because of main items 3 and 4[18]. As for item 6, good for both methods are good, because they use single-carrier transmission. In addition, it is not good for OFDM transmission. As for item 7, we described flexible allocation of bandwidth of SC-FDMA in chapter 3.3. As for item 8, the difference of SC-TDE and OFDM is the processing timing of IFFT.

## 5. Conclusion

Recently, frequency-domain equalization for single-carrier transmission (SC-FDE) technology has been given much attention[14]-[24]. Actually, for the up-link in the enhanced system an SC-FDMA method using SC-FDE and multiple access has been adopted.

In this technical note, we consider single-carrier adaptive equalization using the MMSE criterion in the time-domain and the frequency-domain in a unified way centered on the Wiener Filter, and discussed each feature. Also, we considered the significance of the CP and explained the basic principle of the SC-FDE and SC-FDMA in an organized and systematic way. Moreover, we pointed out relationship between SC-TDE and SC-FDE Wiener solutions and the physical meaning of the Wiener solution in SC-FDE.

Therefore, we could show useful information and pointers when we design new communication systems using SC-FDE. This information is useful especially for when we want to replace existing SC-TDE technology with SC-FDE technology.

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## Appendix

1. Derivation of the optimal tap gains $W_{freq\_opt}$ in the frequency domain shown as equation (16)

First, the MSME of the error signal is as follows.

$$E\left[|e_n|^2\right] = \frac{1}{N^2} E\left[\sum_{k=0}^{N-1} |W_k Y_k - X_k|^2\right]$$
$$= \frac{1}{N^2}\sum_{k=0}^{N-1}\Big(W_k^* W_k E\left[|Y_k|^2\right] - W_k^* E\left[Y_k^* X_k\right]$$
$$- W_k^* E\left[Y_k X_k^*\right] + E\left[|X_k|^2\right]\Big). \quad \cdots (25)$$

To derive $W_{freq\_opt}$, we differentiate using the complex gradient operator. So, for the complex number $W=W_{real}+jW_{imag}$, the partial differentiation of W is as follows.

$$\frac{\partial}{\partial W} = \frac{1}{2}\left(\frac{\partial}{\partial W_{real}} - j\frac{\partial}{\partial W_{imag}}\right). \quad \cdots (26)$$

Similarly, for the complex conjugate of $W$, $W^*=W_{real}-jW_{imag}$ the following equation holds.

$$\frac{\partial}{\partial W^*} = \frac{1}{2}\left(\frac{\partial}{\partial W_{real}} + j\frac{\partial}{\partial W_{imag}}\right). \quad \cdots (27)$$

From equation (20) we solve

$$\frac{\partial}{\partial W_k} E\left[|e_n|^2\right] = W_k E\left[|Y_k|^2\right] - E\left[Y_k^* X_k\right] = 0. \quad \cdots (28)$$

and can get the following $W_{freq\_opt}$.

$$W_{freq\_opt} = \frac{E\left[Y_k^* X_k\right]}{E\left[|Y_k|^2\right]}$$
$$= \frac{E\left[\left(H_k^* X_k^* + N_k^*\right)X_k\right]}{E\left[\left(H_k^* X_k^* + N_k^*\right)\left(H_k X_k + N_k\right)\right]}$$
$$= \frac{H_k^* E\left[|X_k|^2\right]}{|H_k|^2 E\left[|X_k|^2\right] + E\left[|N_k|^2\right]}$$
$$= \frac{H_k^*}{|H_k|^2 + 1/\gamma_k}. \quad \cdots (16)$$

2. Derivation example of a circulant matrix

We consider the following transmitted signal $x$.

$$x = \begin{bmatrix} x_0 & x_1 & x_2 & x_3 \end{bmatrix}. \quad \cdots (29)$$

We insert a CP ,which corresponds to two symbols, in the signal x.

$$\mathbf{X} = \begin{bmatrix} x_2 & x_3 & x_0 & x_1 & x_2 & x_3 \end{bmatrix}. \quad \cdots (30)$$

When the impulse response of the channel is given by $h=[h_1, h_2]$ and $x$ is transmitted for only one block, the received signal $X$ is as follows.

$$\mathbf{Y} = \mathbf{HX}$$
$$\begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} = \begin{pmatrix} h_0 & 0 & 0 & 0 & 0 & 0 \\ h_1 & h_0 & 0 & 0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}. \quad \cdots (31)$$

Here, the CP is eliminated.

$$\begin{pmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \end{pmatrix} = \begin{pmatrix} 0 & h_1 & h_0 & 0 & 0 & 0 \\ 0 & 0 & h_1 & h_0 & 0 & 0 \\ 0 & 0 & 0 & h_1 & h_0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & h_0 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \\ x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}$$
$$= \begin{pmatrix} h_0 & 0 & 0 & h_1 \\ h_1 & h_0 & 0 & 0 \\ 0 & h_1 & h_0 & 0 \\ 0 & 0 & h_1 & h_0 \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix}. \quad \cdots (32)$$

Therefore the circulant matrix could be obtained.

3. Derivation of the relationship between SC-TDE and SC-FDE Wiener solutions (24)

As for equation (16), a similar equation is derived for a continuous-time system, too. That is, the following equation is satisfied.

$$W_{freq\_opt}(f) = \frac{H^*(f)}{|H(f)|^2 + \left[S_s(f)/S_n(f)\right]^{-1}} \quad \cdots (16)'$$

From equation (16)', equation (24) is derived using equation (3).

$$W_{freq\_opt}(f) = \frac{H^*(f)}{|H(f)|^2 + \left[S_s(f)/S_n(f)\right]^{-1}}$$
$$= \frac{S_s(f)H^*(f)}{|H(f)|^2 S_s(f) + S_n(f)}$$
$$= \frac{|H(f)|^2 S_s(f)}{|H(f)|^2 S_s(f) + S_n(f)} \frac{H^*(f)}{|H(f)|^2}$$
$$= \frac{S_x(f)}{S_x(f) + S_n(f)} \frac{H^*(f)}{|H(f)|^2}$$
$$= \frac{H^*(f)}{|H(f)|^2} W_{time\_opt}(f). \quad \cdots (24)$$

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