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Performance of GMSK and Reed-Solomon Code Combinations

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Abstract— This paper examines a coded Gaussian Minimum Shift Keying (GMSK) system which uses Reed-solomon (RS) codes in Additive White Gaussian Noise channels.The performance of GMSK and RS code combinations is compared with the constraint that the transmission bandwidth is constant. The coding gains were obtained using simulations and the best combination of GMSK and RS codes was found.

Keywords—GMSK, Reed-Solomon codes, coding gain

1 Introduction

Since the GMSK modulation method was first proposed by K.Murota and K.Hirade [1], it has gained a great deal of attention and has been adopted as the modulation scheme of cellular systems such as GSM and DCS1800 due to its compact power spectral density and excellent error performance. Although the performance of GMSK has been analyzed by several researchers, coding for GMSK has received little attention. In our previous research, optimal combinations of GMSK modulation and BCH coding for a given transmission bandwidth were found [2][3]. This paper focuses on GMSK and Reed-Solomon(RS) coding.

Error control codes insert redundancy into the transmitted data stream so that the receiver can correct errors that occur during transmission. Therefore, the bit interval of the coded bits must be shortened in order to keep the information transmission rate constant. A shorter bit interval results in a larger transmission bandwidth. In order to keep the bandwidth of the coded system the same as that of the uncoded system, the modulator used in the coded system must adopt a smaller value of B_b, which is introduced in the next section, than that of the modulator used in the uncoded system.

This paper is organized as follows. The next section gives an overview of the system including a description of GMSK modulation and RS codes. Section 3 gives our approach to the bandwidth allocation problem. Simulation results for GMSK modulation with different Reed-Solomon coding rates are presented in Section 4, and the conclusions are given in Section 5.

2 System Overview

2.1 System Model

We consider the GMSK modulation and Reed-Solomon coding system shown in Figure 1. The performance of various combinations of GMSK and RS codes is evaluated with the constraint that the total system bandwidth is constant. The bandwidth of GMSK can be easily controlled by the parameter B_b. The uncoded systems is also evaluated to serve as a benchmark.

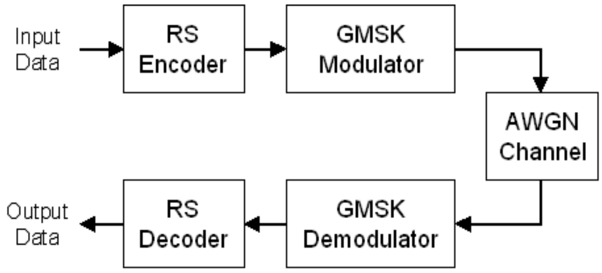

Figure 1: System Model

2.2 GMSK Modulation

GMSK, as its name suggests, is based on MSK and was developed to improve the spectral properties of MSK by using a premodulation Gaussian filter. The filter impulse response is expressed as:

$$h(t) = \frac{1}{\sqrt{2\pi}\sigma T} \exp\left(\frac{-t^2}{2\sigma^2 T^2}\right) \quad (1)$$

where $\sigma = \frac{\sqrt{\ln(2)}}{2\pi B_b T}$

The Gaussian filter is characterized by its B_bT product (B_b is the -3dB bandwidth of the Gaussian prefilter and T is the symbol period.) The lower the B_bT product,the narrower the modulation bandwidth. In this paper, we use $B_bT = 1.0$ and $B_bT = 0.5$ for the uncoded system. For transmission in an AWGN channel, the bit error rate of GMSK is given by

$$p = \frac{1}{2} erfc\left(\sqrt{\frac{d_{min}^2 E}{2N_0}}\right) \quad (2)$$

where d_{min} is the normalized minimum Euclidean distance between the signal representing "0" and the signal representing "1", E is the energy per transmitted bit and $N_0/2$ is the power spectral density of the AWGN.

2.3 Reed-Solomon codes

Block codes used in practical applications today are linear cyclic codes, since these codes are easier to implement. Block coding schemes involve dividing the input data into k-bit blocks and then mapping each k-bit block into an n-bit block called a codeword, where $n > k$ in the encoding process. $(n-k)$ check bits are added to each k-bit block. The ratio $r = k/n$ is called the code rate. Binary BCH (Bose Chaudhuri

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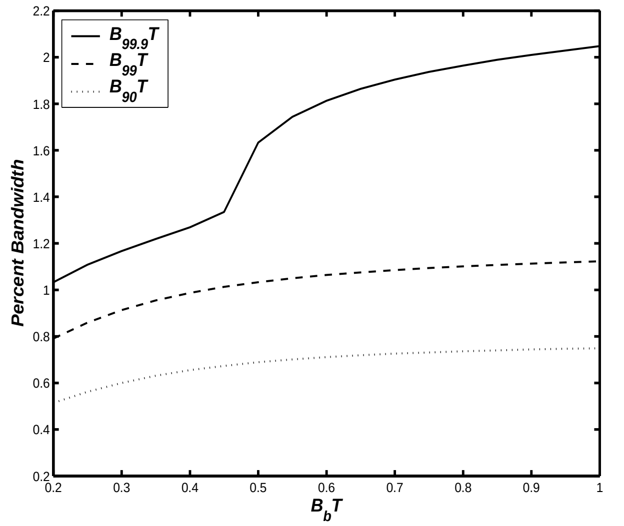

Figure 2: Percent power containment bandwidths for GMSK

Hocquenghem) codes and nonbinary RS codes are two kinds of widely used linear cyclic block codes.

RS codes are nonbinary BCH codes, which are useful for correcting burst errors. The data is partitioned into symbols of m bits, and each symbol is processed as one unit both by the encoder and decoder. RS codes satisfy: $n \leq 2^{m-1}$ and $n-k \geq 2t$, where t is the number of correctable symbol errors. The RS decoded symbol error probability, P_E, in terms of the channel symbol error probability, p, can be written as follows [4]:

$$P_E \approx \frac{1}{2^m-1} \sum_{j=t+1}^{2^m-1} j \binom{2^m-1}{j} p^j (1-p)^{2^m-1-j} \quad (3)$$

Since RS code parameters have a significant effect on coding performance, RS codes with different rates are tested.

3 Performance Evaluation

In order to determine what combination of coding rate and B_b for the coded system results in the same bandwidth as the uncoded system, we must choose the measure of bandwidth. In this paper, we use the percent power containment bandwidth, denoted by B_x and defined as the bandwidth which contains $x\%$ of the signal power. B_{90}, B_{99} and $B_{99.9}$ are plotted in Figure 2. $B_{99.9}$ for GMSK, which is the bandwidth that contains 99.9% of the signal power, is used in the simulations. In (2) for the uncoded system, the value of E is E_b, which is the energy per transmitted information bit. For the coded system, the value of E is set to be rE_b, since the energy for the coded bits is spread among the more numerous coded bits. This allows a fair comparison to be made between the uncoded and coded systems.

It is complicated to compute the bit error probability P_b by using (2) and (3) because RS codes are nonbinary codes, so we use MATLAB. The simulation model is shown in Figure 3. In the simulations the following parameters were used.

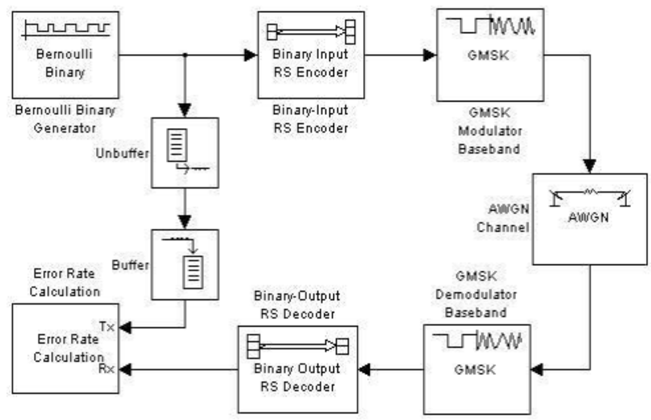

Figure 3: Simulation Model

Table 1: B_bT and RS code parameter combinations that result in systems with equal bandwidths

n=63				
	k			
uncoded	57	53	49	41
0.5	0.47	0.46	0.4	0.22
1.0	0.64	0.54	0.49	0.45
n=127				
	k			
uncoded	103	99	87	79
0.5	0.45	0.4	0.26	0.17
1.0	0.51	0.49	0.46	0.4

- Input data 100000 bits
- RS codeword length: 63, 127

4 Results

In this section, the parameters used in the simulation model are discussed. Simulation results are also presented. BER (Bit Error Rates) for different systems were evaluated using simulations. The code parameters used were RS$(127, k)$ and RS$(63, k)$. In order to ensure the bandwidths of the coded and uncoded systems are the same, we have to calculate the value of B_bT. For example, if we set the value of $B_bT = a$ for the uncoded system, then $B_{99.9}T = A$ from Figure 2. When the RS coding rate is r, the coded $B_{99.9}T = B$ is calculated from: $B = A \times r$. The corresponding value of B_bT is found from Figure 2. The parameters used in the simulations are shown in Table 1.

The simulation results are shown in Figures 4–7. From Figure 4, we can intuitively find that as the code rate decreases, in other words as the value of B_bT decreases, the bit error probability decreases. When $B_bT = 0.4$(RS(63,49)) P_b reaches a minimum value. However, as the code rate decreases further, P_b increases instead, as the curve for $B_bT = 0.22$ shows. The reason is that the code rate r decreases but the

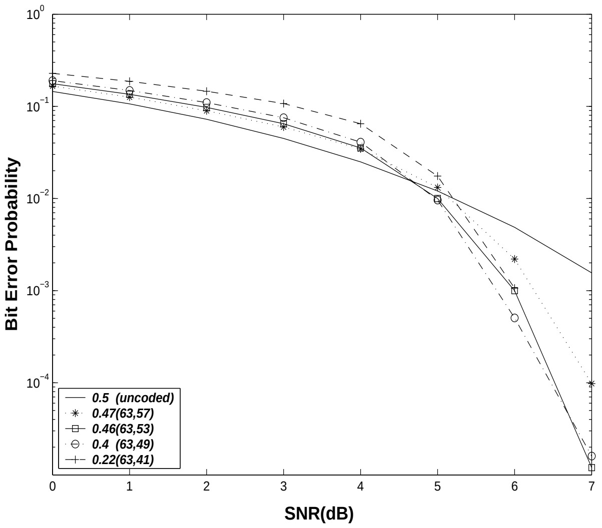

Figure 4: Performance for GMSK-RS code combinations with the same bandwidth as an uncoded system with $B_bT = 0.5$

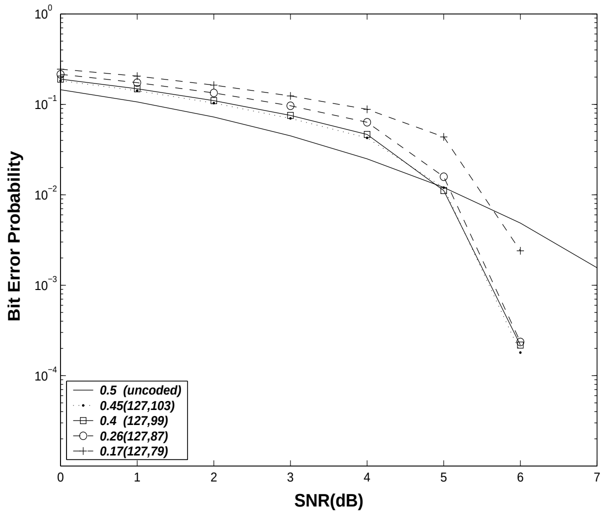

Figure 6: Performance for GMSK-RS code combinations with the same bandwidth as an uncoded system with $B_bT = 0.5$

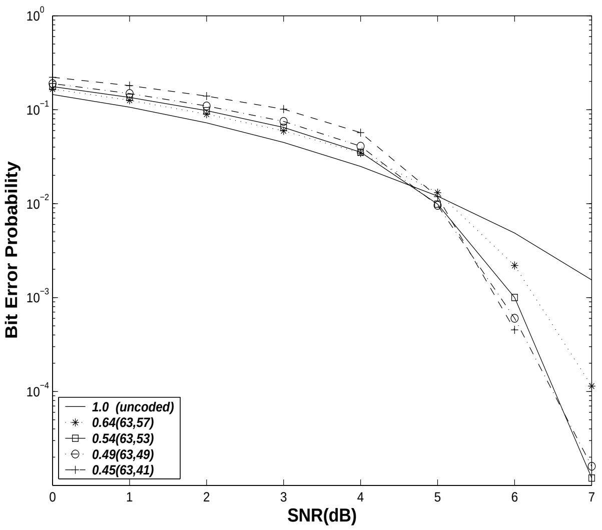

Figure 5: Performance for GMSK-RS code combinations with the same bandwidth as an uncoded system with $B_bT = 1.0$

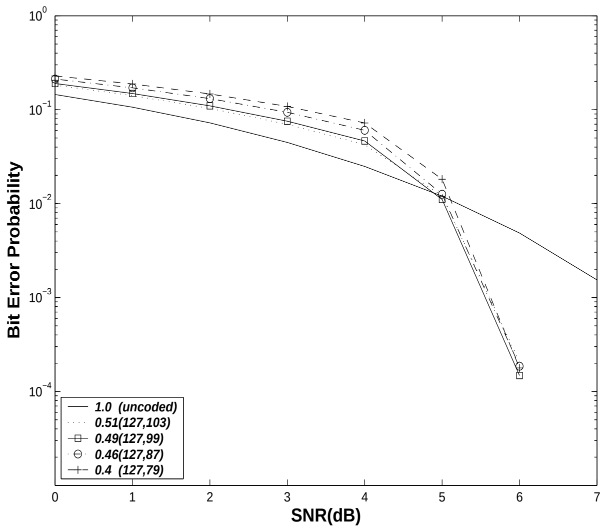

Figure 7: Performance for GMSK-RS code combinations with the same bandwidth as an uncoded system with $B_bT = 1.0$

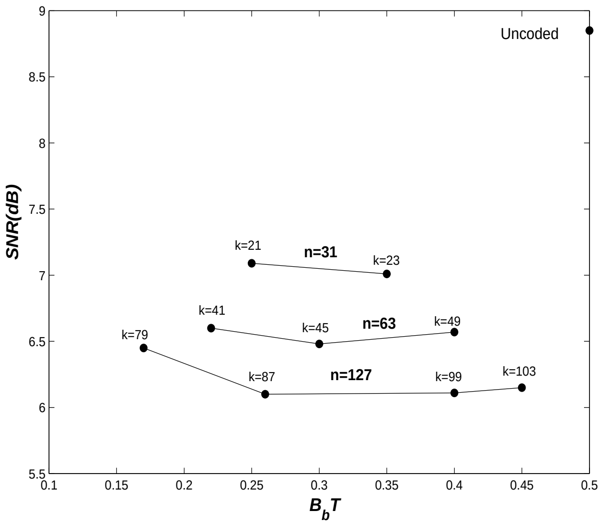

Figure 8: SNR required to achieve a bit error of 10^{-4}

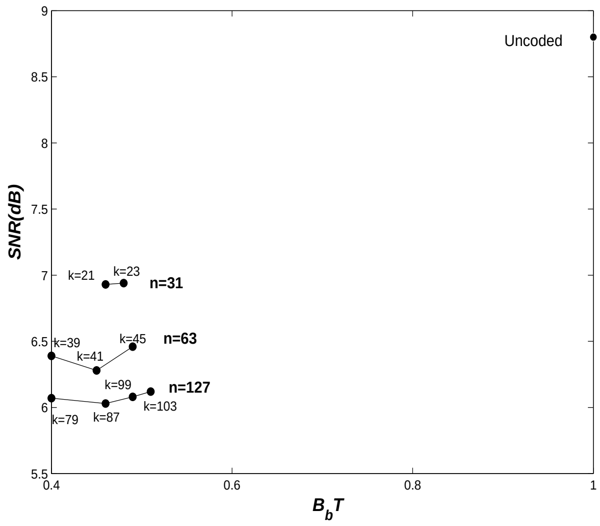

Figure 9: SNR required to achieve a bit error of 10^{-4}

bit error rate of GMSK p increases, so the net result of decreasing r depends on how much p increases in response to a decrease in r. When p increases more than r decreases P_b becomes worse. Figure 5, 6 and 7 show similar results for different system parameters.

In Figures 8 and 9, we show the SNR required to achieve $P_b = 10^{-4}$ when the value of $B_bT = 0.5$ and $B_bT = 1.0$ for the uncoded system. A smaller value of B_bT means a smaller code rate. As the figures show, as the code rate is decreased, the performance improves until a minimum is reached and then worsens. The optimal combination of coding and modulation for a given total transmission bandwidth can be found from these figures. The (63,45) RS shows the maximum gain ($2.4dB$) achieved over the uncoded system ($B_bT = 0.5$). For n=127, the (127,87) RS has the largest gain at $2.75dB$.

From Figure 9, we can see the (31,21) (63,41) (127,87) RS codes achieved the lowest P_b with a code rate of about 0.67, i.e., the code rate at which the best performance is achieved is approximately 2/3. This means that using 2/3 of the bandwidth for modulation and 1/3 of the bandwidth for coding is the optimal balance between modulation and coding for GMSK when $B_{99.9}T = 2.048$. When $B_{99.9}T = 1.633$, i.e., when $B_bT = 0.5$, the optimal balance is 5/7 for modulation and 2/7 for coding. We also can see that the smaller the bandwidth is, the more bandwidth should be used for modulation.

5 Conclusions

In this paper, we examined the performance of coded GMSK systems which employ RS channel coding. Under a constant bandwidth constraint, we optimized the combination of coding and modulation.

For the systems we considered, our results show the bit error rate probability performance can be improved with the same bandwidth as an uncoded system. For a given bandwidth, it is beneficial to use a smaller code rate. However, when the code rate decreases to a certain value, the system performance decreases. For this reason the optimal code rate was found to be a function of the total system bandwidth.

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